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STUDY 32, Vol 2

A Study of Aviation Resources and Readiness Relationships

Volume II

A Ready Hour Production Function for Naval Aviation

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CENTER FOR NAVAL ANALYSES

Institute of Naval Studies

Study 32

A STUDY OF AVIATION RESOURCES AND READINESS RELATIONSHIPS

VOLUME II

A READY-HOUR PRODUCTION FUNCTION FOR NAVAL AVIATION

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ABSTRACT

This volume examines the relationship between aircraft readiness and the aircraft, maintenance labor, and spare parts available at the squadron level. A production function, which shows the relationship between aviation resource use and squadron readiness, was estimated for the A-7B, CH-53, S-2E, F-4B, and TA-4F from 3M data. The production functions and derived cost functions for the F-4B, TA-4F, and CH-53 were used to determine the mix of aircraft, maintenance labor, and spare parts that will maximize the level of readiness for a given budget. Finally, the relationship between the NORS rate and the investment in spare parts is estimated for these type/model/series of aircraft. The methodology employed in this study can be applied to other aircraft types.

Volume III of this study develops a model of the Navy's aviation resupply system and examines various means of increasing the effectiveness of the system using two measures: decreasing the length of time between submitting a requisition and receiving the part, and maximizing the percentage of requisitions filled by a certain day.

Volume I is a summary volume and contains a description of the project, the methodologies used, and the principal conclusions and recommendations.

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SYNOPSIS

This study is concerned with the relationship between aircraft readiness and the aircraft, maintenance labor, and spare parts available at the squadron level. Our objective was to develop and apply a practical method that could determine:

1. How aircraft readiness is affected by changes in the usage of spare parts, maintenance labor and aircraft; and

2. How to combine these resources to get the highest level of aircraft readiness for a given expenditure on them.

METHODOLOGY

The basic elements of the analysis were the estimation of a production function from data on readiness and resource usage and the derivation of a cost function from a non-linear optimization model for each type/model/series studied. The production function shows the relationship between aviation resource use and squadron readiness. The cost function analysis reveals how many hours could be achieved for a given budget, and the resulting optimal mix of primary resources. Only the estimated production function is necessary to determine the relationship between the NORS rate and spares used.

The squadron was chosen as the basic subject of analysis because it is the primary unit charged with combining Naval aviation resources into tactically required output. The measure of aircraft readiness used in the study is aircraft ready hours, defined as follows:

Ready hours (RH) = Total custodial hours

- (Time in NORS condition)

- (Time in NORM condition).

The ready-hour measure was chosen because it reflects only the availability of operational aircraft. Flight hours is sometimes suggested as another measure of squadron output, but flight hours represent a combination of the availability of aircraft and the tactical requirements for available aircraft. Ready hours seem therefore, a more accurate measure of the effectiveness of the aviation supply and logistics system.

In constructing the ready-hour measure, no attempt was made to differentiate among various categories of NORS and NORM conditions. Such differentiation requires the construction of an "essentiality" index, an analytical problem beyond the scope of the present study. In this study, an aircraft is considered to be unready if, for any reason, it is unable to fully perform its primary mission.

The procedures used in this study to estimate the parameters of the ready-hour production function belong to the general class of non-linear, iterative techniques. The problems involved in econometrically estimating parameters of a production function are mathematically complex. Certain points can be noted

here, however. If one assumes that the squadron inputs have been purchased in an economically competitive market, then standard linear regression can be used to estimate the parameters of the production function. There is, however, no strong reason to believe that this assumption is descriptive of the Navy logistic system. Therefore, a non-linear, iterative technique, which does not rely upon this assumption, was employed.

Five types of aircraft were chosen for empirical estimation of the type described above:

1. The A-7B, an attack aircraft.
2. The F-4B, a fighter.
3. The S-2E, an anti-submarine aircraft.
4. The CH-53, a helicopter.
5. The TA-4F, an attack trainer.

Data was collected from the Navy Maintenance Material Management System (3M) on the usage of these aircraft, maintenance man-hours, and spare parts at the squadron level and on squadron readiness. Each type/model/series of aircraft in the inventory will be expected to have a unique production function. This cross-section of types, however, is sufficient to demonstrate the usefulness of the production function approach and to draw some general conclusions regarding logistics support for these types of aircraft. Data was stratified by location and deployment of each type aircraft in the following breakdown: all squadrons, Atlantic squadrons, Pacific squadrons, and training squadrons. Production functions were estimated in each of these different operating environments.

The econometric estimation of a ready-hour production function is the first step in the determination of the economically most efficient combination of squadron inputs to produce a given level of ready hours. The ready-hour production function merely reveals the technological relationships between resource usage and the production of ready hours. As useful as this information is, the costs or prices of the resources must be introduced in order to make meaningful statements about how to allocate various budgets among the support resources of aircraft, maintenance man-hours, and spare parts. The problem is one of determining the maximum number of ready-hours attainable at the squadron level, given a budget constraint or fixed funding level on the usage of aircraft, maintenance labor, and spare parts. Given the squadron ready-hour production function, estimated by the procedures discussed above, and the budget constraint on the squadron's resources, it is then possible to formulate the optimization problem:

Maximize ready hours, subject to a given budget constraint on resource usage.

The solution to this problem yields a relation between the budget for resource usage and the level of squadron output. In addition, the optimal combination of inputs may be identified for each output level.

MAIN RESULTS

- The results indicate that this methodology can be valuable in examining squadron operation. Much of the observed variance in squadron readiness is explained as a function of resource usage.

- Given current operating practices, the results suggest a need for increased spare parts support. Of course, since the value of this support depends upon the particular items being provided to the squadrons, a more efficient inventory management system could lead to even greater readiness. Until such practices are developed and implemented, however, we find that a larger proportion of the budget should be allocated to the spare parts support category.

For more detailed results of the analysis, please see the first section of the main text.

INTRODUCTION, CONCLUSIONS AND RECOMMENDATIONS

BACKGROUND

This study addresses the problem of determination of the relationship between the usage of various primary inputs and the readiness of Naval aviation squadrons. At the squadron level--the basic operating unit of Naval aviation--these primary inputs are readily identified. In order to produce output (ready aircraft), the squadron employs a mix of aircraft, maintenance labor, spare parts and various other inputs. These inputs can be substituted in various ways to produce a given level of ready aircraft. For example, greater maintenance capabilities may reduce the total number of aircraft required by the squadron. Or, more rapid resupply capabilities may reduce the number of on-board spares required. Further, these inputs can often be complementary, as well as substitutable. The addition of more on-board spares may be nearly valueless if the maintenance capabilities of the squadron are already employed at capacity.

A fundamental objective of the Navy Logistics system, therefore, is to furnish the required levels of these various inputs to the productive units charged with carrying out the various tactical missions for which they were designed. Furthermore, recognizing the variety of ways in which these inputs can be combined to produce a given level of readiness, the logistics system must have as an objective some criterion of efficiency in choosing among the tactically acceptable alternatives. The criterion of minimum cost has traditionally been accepted in this respect, although other possibilities, such as flexibility, have been employed at times.

An important part of the logistics system is spare parts support. Two aspects of the general problem of spare parts management have received special attention, as a means of providing decision procedures leading to a minimum cost attainment of a given objective function. The first has been inventory theory, dealing with the problems of determining optimal stockage and re-order rules under various operating environments. The second has been the general class of reliability models. Two specific problems have been addressed: the effects of standby spare systems on the operation of a complex system, and the effects of repair and maintenance on the operation of a complex system.

Other aspects of the spare parts management problem that have received study include the effects of cannibalization policies on the reliability of various operating systems and the determination of the optimal combination of resources in a sequential flow system for the transportation of spare parts.

In general, all of these studies have treated the spare parts problem as a self-contained system, either ignoring the interaction with other aspects of the total logistics system or taking it as given. This approach, of course, cannot be criticized; it has lead to many valuable models and decision rules required for the efficient daily operation of this large system.

However, as noted earlier, spare parts provide only one input to the productive units being served by the total logistics system. Other inputs -- aircraft and maintenance labor, for example, are required along with spare parts in order for these productive units to operate. The objective of this study is to determine the ways in which these various inputs interact in order to produce the final output for which they were furnished. Once this determination is made, the criterion of cost minimization may be employed to suggest efficient combinations of these broad categories of inputs to use in the production of readiness.

These objectives, of course, are different from those of the typical inventory or reliability model, primarily in the level of aggregation being considered. In considering spare parts as a single, distinct input to the productive process under study, no consideration can be given to part-by-part tradeoffs, as is usually done in inventory and reliability models. Nevertheless, the ability to consider tradeoffs among the broadly defined logistics categories of aircraft, maintenance labor, and spare parts can be of great value to the logistics system manager.

METHODOLOGY

The objective of determining the relation between the usage of these inputs and the output of readiness suggests the use of a production function, which may be defined as a function relating the level of output produced to the levels of the various inputs employed to produce that output. The concept of a production function has been extensively used in a variety of economic applications. Here it is used to investigate the empirical relation between the production of readiness and the use of aircraft, maintenance labor, and spare parts at the squadron level.

In order to conduct the analysis, some measure of the squadron's output must be defined. Two come to mind -- flight hours and ready hours. The Ready hours measure may be defined as:

Ready hours = Total custodial hours
- (Time in NORS condition)
- (Time in NORM condition)

We chose ready hours because they reflect only the availability of operational aircraft; flight hours represent the combination of the availability of aircraft and

the tactical requirements for available aircraft. It seemed, that is, that ready hours are a more nearly accurate measure of the effectiveness of the logistics system. In constructing this measure, no differentiation was made among the various categories of NORS and NORM conditions, due to the difficulties in constructing an essentiality index. Therefore, an aircraft is considered to be unready if, for any reason, it is unable to fully perform its primary mission.

Using ready hours as a measure of the squadron's production, our objective then is to determine how the various logistics inputs used by the squadron -- aircraft, spare parts, and maintenance labor -- interact to lead to the observed production of ready hours. The maintenance labor input to the production is only the subject of total maintenance labor employed in remedial actions related to squadron aircraft readiness (i.e., the removal, repair, and installation of spare parts). Labor employed in activities such as flight preparation is not included in this analysis.

Five types of aircraft were chosen for study to obtain a broad cross-section of the types of aircraft supported by the Navy logistics system: an attack aircraft (the A-7B), a fighter (the F-4B), an anti-submarine aircraft (the S-2E), a helicopter (the CH-53), and an attack trainer (the TA-4F). Data was collected from the Navy Maintenance Material Management System (3M) for squadrons employing these five types of aircraft. While it is recognized that each type/model/series of aircraft in the Navy inventory could be expected to exhibit a unique production function, this cross-section serves both to demonstrate the methodology of the analysis and to lead to some general conclusions regarding logistics support for these types of systems.

Having estimated the parameters of the algebraic form of the production function from the data collected on these five aircraft types, many questions regarding the effect of logistics decisions on the production of ready hours can immediately be answered. The effects of changes in the level of logistics support in any support category on the production of ready hours can be estimated from the production function. Furthermore, the effects of proportionate increases or decreases in the level of support of all categories can be measured. Finally, by separating the data for the five aircraft by location and deployment, the effectiveness of the various types of logistics support in different operating environments can be estimated. All of these facts can be of significant importance to the logistics planner.

The final output of the analysis addresses the question of determining the quantities of the various inputs that should be provided to achieve a certain readiness level at minimum cost. Once the production function is estimated, the various alternative combinations of inputs that lead to the same level of readiness can be identified. By determining the costs of providing these various levels of inputs, the minimum cost combination can be chosen.

This optimization procedure can be employed for the many different levels of readiness of interest to the tactical planner. The output resulting from this procedure is a cost function, which may be defined as a function giving the minimum total cost of input usage required to support any given level of readiness desired. The cost function is found by determining the optimal allocation of the budget among resource categories, and will tell us the minimum budget for input usage required to support a desired ready-hour level. Given this result, it is then possible to determine the minimum cost tradeoff between initial stockage and resupply, as various combinations of stockage and resupply can provide the desired level of spares usage. In addition to this optimum level, we could determine the effects on ready-hour production and on cost from any decision which causes a change in the level of inputs used by the squadron.

RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

It has been shown that the applied model developed in this study can be used to determine the following things:

1. How aircraft readiness is affected by changes in the usage of spares and other primary resource inputs.

2. How to combine maintenance man-hours, aircraft, and spare parts to get the highest aircraft readiness for a given budget.

The capabilities of the model are demonstrated by its application to the F-4B, TA-4F, and CH-53. The results of these applications illustrate the model's value as a planning tool. The following major analytical results were derived from the aircraft studied:

1. For the current estimated budget for the F-4B, there should be, from a logistics point of view, one less aircraft per squadron, a 7 percent decrease in man-hours expended, and about a 25 percent increase in spare parts used. If this resource mix were used, ready hours per squadron would increase by approximately 41 percent and ready hours per aircraft would increase by 54 percent.

2. The optimal results for the CH-53 indicates that the logistic size of the squadron should be reduced by 2 helicopters, maintenance man-hours should be more than doubled, and spares usage increased by about 66 percent. This would increase ready hours for a squadron of CH-53's by about 8 percent of the same total cost.

3. Although the maximum ready hours for the TA-4F squadrons are very close to the average value we have observed at the same total cost, the optimal allocation of resources indicates that maintenance man-hours should be reduced and that spares usage should be increased by 11 percent.

4. Assuming a proportional relationship between spare parts used at the squadron level and dollars budgeted for spares, changes in the spares budget can be related to changes in ready hours and NORS rates by type/model/series of aircraft. Given the present system, a 10 percent increase in spares usage will reduce the NORS rate for the F-4B about 16 percent; for the CH-53 by about 2.3 percent; and for the TA-4F by about 8.4 percent.

Two primary conclusions are drawn from the above results.

- First, if current total resource budgets are maintained in the future, a larger percentage of those budgets should be allocated to the spares categories and a smaller percentage to aircraft and maintenance.

- Second, in all cases, the results indicate the need for more spare parts.

- The results just described could be determined for any specified budget. This would make it possible to determine how to efficiently allocate total budget cuts for a type/model/series among the aviation resource categories, i.e., to allocate budget reductions in order to achieve the smallest reduction in ready hours. Arbitrary cuts in specific budget categories could reduce the level of effectiveness of the aircraft far below what could be achieved if the burden of the cut was efficiently allocated.

5. There are a large number of possible extensions of the production function technique. Further stratification of the input resource categories could be of even greater use in planning, as trade-offs among various categories of maintenance labor and spare parts could then be considered. Development of an output measure which differentiated among different levels of readiness could be another extension. Finally, extension of the technique to other aircraft types would provide additional input to the budget planning process.

THE THEORY OF PRODUCTION FUNCTIONS

INTRODUCTION

In this section of the report we develop the concept of a production function and determine the properties of the various algebraic forms of the production function employed in this analysis. While our study considers more than two inputs, the discussion for simplicity is restricted to a process employing only two inputs in the production of a single output. The results discussed here are readily generalized to the higher dimension case.

PROPERTIES OF A PRODUCTION FUNCTION

We define a production function as a function associating a level of output, X, with each combination of input levels, x_1, and x_2:

$$X = X(x_1, x_2)$$

Intuitively, we would expect such a production function to have the following properties:

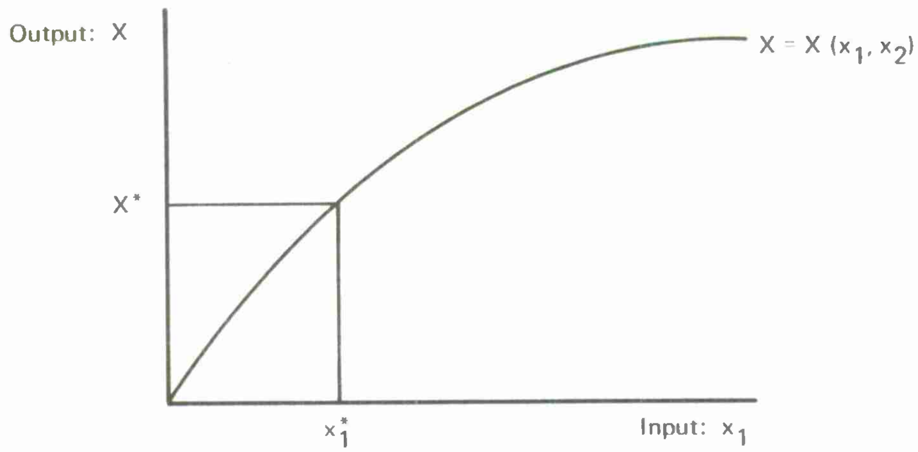
(1) An increase in the level of any input should produce an increase in the level of output.

(2) Subsequent increases in the level of any one input, holding all other inputs constant, should produce smaller and smaller absolute increases in the level of output.

(3) The marginal increase in output resulting from an increase in any one input will be greater if other inputs are also increased.

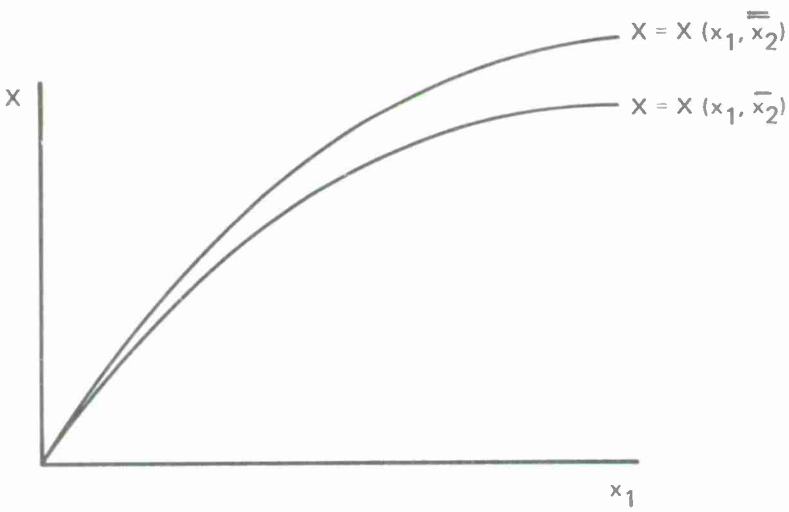
(4) Many different combinations of inputs can be used to produce the same level of output.

The first two properties are shown in the figure below:



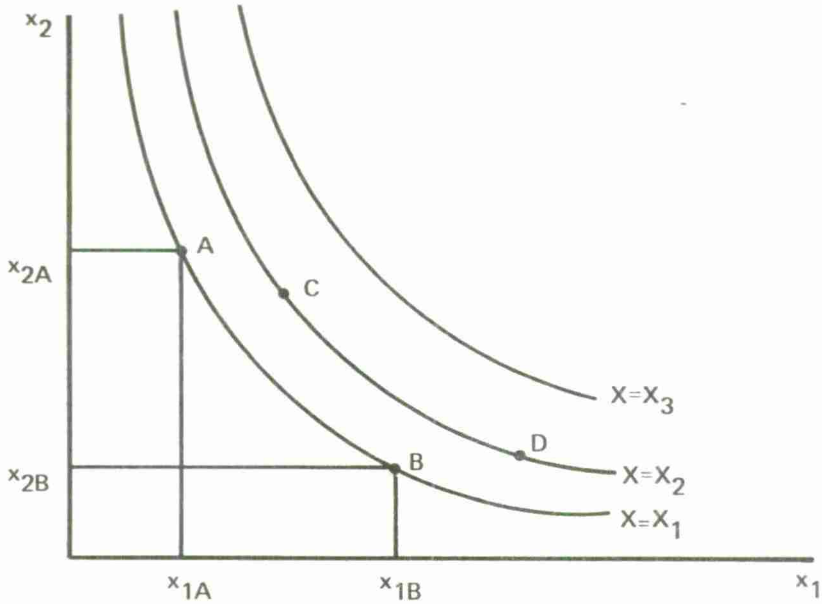
For any constant level of the second input (say, $x_2 = \bar{x}_2$), the curve shows the level of output resulting from the use of any particular level of input x_1. For example, using a level x^*_1 (along with $\bar{x}_2$) results in an output level X^*. Note also that higher levels of x_1 always lead to a higher level of output being produced, although subsequent increases in x_1 produce smaller and smaller increases in output.

The third property, that the marginal increase in output resulting from an increase in any one input will be greater if other inputs are also increased, is shown in the figure below for a second level of x_2, say $\bar{\bar{x}}_2$, where $\bar{\bar{x}}_2 > \bar{x}_2$.



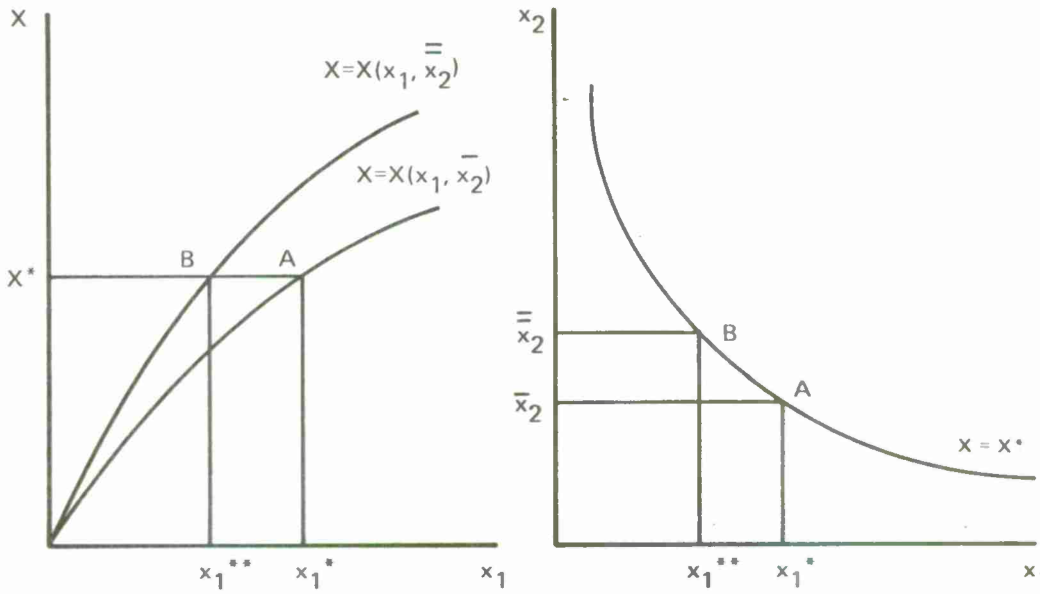
We see that any level of x_1 results in a higher level of output when combined with $\bar{\bar{x}}_2$ than when combined with $\bar{x}_2$.

The fourth intuitive property, that many different combinations of inputs can be used to produce the same level of output, may also be displayed. In this case, the axes are the levels of the two inputs being used. The various curves ($X = X_1$, $X = X_2$, etc.) represent the level of output resulting from using the associated levels of the two inputs. Each curve represents a given, constant level of output, and each point on such a curve represents a combination of inputs leading to the same level of outputs.



The points A (representing input levels of x_{1A} and x_{2A}) and B (representing input levels x_{1B} and x_{2B}) both lead to the production of X_1 units of output, the points C and D both lead to the production of X_2 units of output, and so forth. These curves are referred to as isoquants, or equal product curves. Every point on an isoquant represents an input combination which leads to the same level of output production.

The first and third figures above are really different ways of presenting the same facts, as can be seen in the following figure.



Consider the output level X^*, which we see from the figure on the left results from either the input combination $(x_1^*, \bar{x}_2)$ or from the input combination $(x_1^{**}, \bar{\bar{x}}_2)$. Observing the isoquant $X = X^*$ on the right, we note that these same input combinations are represented as points on the X^* isoquant.

The first three of these intuitively desirable properties can also be expressed mathematically:

(1') $$\frac{\partial X}{\partial x_i} > 0 \text{ for all } i ;$$

(2') $$\frac{\partial^2 X}{\partial x_i^2} < 0 \text{ for all } i ;$$

(3') $$\frac{\partial^2 X}{\partial x_i \partial x_j} > 0 \text{ for all } i, j \text{ such that } i \neq j .$$

The fourth property, suggesting the concept of an isoquant, is defined as the set of all possible combinations of inputs which can be used to produce a given level of output. A mathematical definition of an isoquant could be written:

(4') $$dX = \frac{\partial X}{\partial x_1} dx_1 + \frac{\partial X}{\partial x_2} dx_2 = 0 .$$

That is, the change in output along an isoquant, dX, resulting from changes in the input levels, dx_1 and dx_2, must be zero.

The Cobb-Douglas Production Function

Having established the desired general properties of a production function, we can now turn to two commonly used algebraic forms of this general production function. The first, and the oldest, commonly used form is the Cobb-Douglas (C-D) Production Function:

$$X = M x_1^{\alpha} x_2^{\beta} ,$$

where $M > 0$, $0 < \alpha, \beta < 1$.

This function is homogeneous of degree $\alpha + \beta$:

$$X = X(\lambda x_1, \lambda x_2)$$

$$= M(\lambda x_1)^{\alpha}(\lambda x_2)^{\beta}$$

$$= M\lambda^{\alpha} x_1{}^{\alpha} \lambda^{\beta} x_2{}^{\beta}$$

$$= \lambda^{\alpha+\beta} M x_1{}^{\alpha} x_2{}^{\beta}$$

$$= \lambda^{\alpha+\beta} X(x_1, x_2)$$

The concept of homogeneity is important in understanding the effects on output of proportionate increases or decreases in all inputs. For example, a doubling of all inputs would correspond to $\lambda = 2$ in the above equations. If $\alpha + \beta = 1$, then output would increase by $2^1 (= \lambda^{\alpha+\beta})$, or, in other words, it too would double. If $\alpha + \beta < 1$, then a doubling of all inputs would lead to less than a doubling of output; if $\alpha + \beta > 1$, output would more than double in response to a doubling of inputs. This factor will be of use later in the analysis of cost functions.

The Constant Elasticity of Substitution Production Function

The second commonly used algebraic form of the production function is the Constant Elasticity of Substitution (C. E. S.) Production Function:

$$X = M[Ax_1{}^{-\rho} + Bx_2{}^{-\rho}]^{-(1/\rho)},$$

where $M, A, B > 0$, $-1 < \rho < 0$ or $0 < \rho$.

As written, this function is linearly homogeneous:

$$X = X(\lambda x_1, \lambda x_2)$$

$$= M[A(\lambda x_1)^{-\rho} + B(\lambda x_2)^{-\rho}]^{-(1/\rho)}$$

$$= M[A\lambda^{-\rho} x_1{}^{-\rho} + B\lambda^{-\rho} x_2{}^{-\rho}]^{-(1/\rho)}$$

$$= M\,[\lambda^{-\rho}(Ax_1^{-\rho} + Bx_2^{-\rho})]^{-(1/\rho)}$$

$$= M\,(\lambda^{-\rho})^{-(1/\rho)}\,[Ax_1^{-\rho} + Bx_2^{-\rho}]^{-(1/\rho)}$$

$$= \lambda M\,[Ax_1^{-\rho} + Bx_2^{-\rho}]^{-(1/\rho)}$$

$$= \lambda X\,(x_1, x_2)$$

Any proportionate change in all inputs leads to the same proportionate change in output.

An alternative form of this function, homogeneous of degree μ, is

$$X = M\,[Ax_1^{-\rho} + Bx_2^{-\rho}]^{-(\mu/\rho)}\,.$$

We shall use the linearly homogeneous form in the following text, although a more complex form is employed in the empirical analysis.

An alternative and often used algebraic form of the C. E. S. function is given by:

$$X = Y\,[\,\delta x_1^{-\rho} + (1-\delta)\,x_2^{-\rho}]^{-(1/\rho)}$$

The equivalence between the two forms is seen below:

$$X = M(Ax_1^{-\rho} + Bx_2^{-\rho})^{-(1/\rho)}$$

$$= M\,(A+B)^{-(1/\rho)}\left(\frac{1}{(A+B)}\right)^{-(1/\rho)}(Ax_1^{-\rho} + Bx_2^{-\rho})^{-(1/\rho)}$$

$$= M\,(A+B)^{-(1/\rho)}\left(\frac{A}{A+B}\,x_1^{-\rho} + \frac{B}{A+B}\,x_2^{-\rho}\right)^{-(1/\rho)}$$

$$= Y(\,\delta x_1^{-\rho} + (1-\delta)\,x_2^{-\rho})^{-(1/\rho)}\,,$$

$$Y = M(A+B)^{-(1/\rho)} ,$$

$$\delta = \frac{A}{A+B} ,$$

and

$$1 - \delta = \frac{B}{A+B} .$$

We shall use the first form in our exposition here.

Properties of the Cobb-Douglas and Constant Elasticity of Substitution Production Functions

The results of analyzing the properties of these two algebraic forms of the generalized production function are summarized in table 1. All the desired properties are present. The only difficulty is in seeing that $\frac{\partial^2 X}{\partial x_i^2} < 0$ in the C.E.S. case. Using Euler's theorem,

$$X = \frac{\partial x}{\partial x_1} x_1 + \frac{\partial x}{\partial x^2} x_2$$

(which holds for any linearly homogeneous function), the result follows trivially.

Thus the marginal product of any input, which may be defined as the change in output resulting from an increase by one unit of the input, is positive and diminishing for both production functions, and the marginal product of any input increases with an increase in the other input.

Along any isoquant of these two forms of the production function,

$$dX = \frac{\partial X}{\partial x_1} dx_1 + \frac{\partial X}{\partial x_2} dx_2 = 0 .$$

TABLE 1

PROPERTIES OF PRODUCTION FUNCTIONS

		Cobb-Douglas	Constant elasticity-of-substitution
$\frac{\partial X}{\partial x_1}$	=	$\frac{\alpha X}{x_1}$	$AM^{-\rho}\left(\frac{X}{x_1}\right)^{1+\rho}$
$\frac{\partial X}{\partial x_2}$	=	$\frac{\beta X}{x_2}$	$BM^{-\rho}\left(\frac{X}{x_2}\right)^{1+\rho}$
$\frac{\partial^2 X}{\partial x_1^2}$	=	$\frac{\alpha(\alpha-1)X}{x_1^2}$	$AM^{-\rho}(1+\rho)\left(\frac{X}{x_1}\right)^{\rho}\left[\frac{\frac{\partial X}{\partial x_1}x_1-X}{x_1^2}\right]$
$\frac{\partial^2 X}{\partial x_2^2}$	=	$\frac{\beta(\beta-1)X}{x_2^2}$	$BM^{-\rho}(1+\rho)\left(\frac{X}{x_2}\right)^{\rho}\left[\frac{\frac{\partial X}{\partial x_2}x_2-X}{x_2^2}\right]$
$\frac{\partial^2 X}{\partial x_1 \partial x_2} = \frac{\partial^2 X}{\partial x_2 \partial x_1}$	=	$\frac{\alpha\beta X}{x_1 x_2}$	$AB(1+\rho)M^{-2\rho}X^{1+2\rho}(x_1 x_2)^{-(1+\rho)}$

If we define the marginal rate of substitution, r, as the slope of the isoquant,

$$r = \frac{dx_1}{dx_2} ,$$

It follows from substitution of the results summarized in table 1 that

$$r = - \frac{\partial X / \partial x_2}{\partial X / \partial x_1} < 0$$

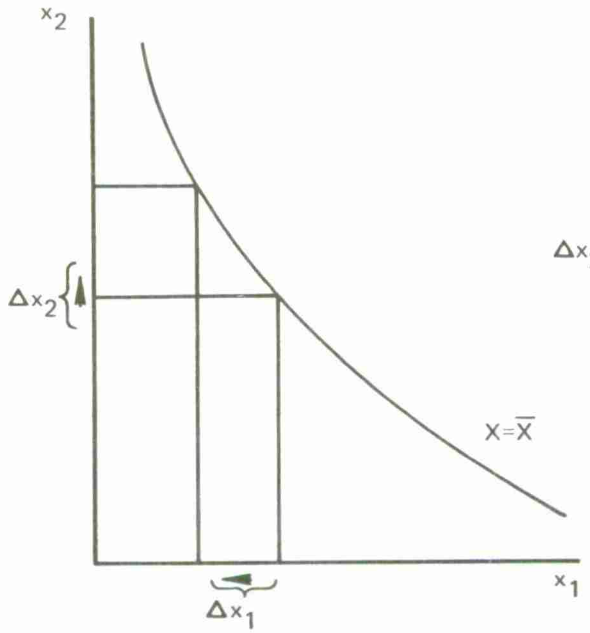
for both the C-D and C. E. S. forms.

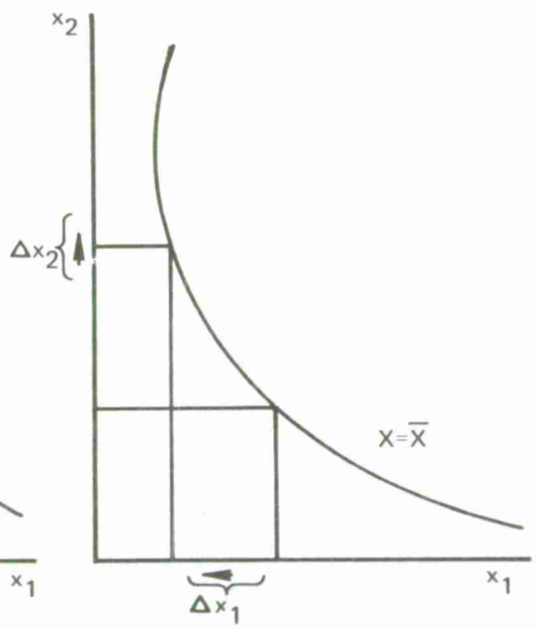
The results of table 1 also reveal that:

$$\frac{d^2 x_1}{dx_2^2} = \frac{-\left(\frac{\partial X}{\partial x_1}\right)\left(\frac{\partial^2 X}{\partial x_2^2}\right) + \left(\frac{\partial X}{\partial x_2}\right)\left(\frac{\partial^2 X}{\partial x_1\, \partial x_2}\right)}{\left(\frac{\partial X}{\partial x_1}\right)^2} > 0$$

for both forms of the production function. Hence the isoquant maps must be of the general forms drawn earlier.

The ability to substitute one input for another is essential to the following analysis. In the two isoquant maps shown below,





consider the decrease in the level of the first input, represented as Δx_1. In order to maintain output constant at a level $\overline{X}$, an increase in the second input of Δx_2 is required. Where a very large increase in x_2 is required to compensate for the loss in x_1, as in the right figure, we might consider the two inputs poor substitutes for one another.

Since the rate of substitution changes as we move along the isoquant, we can write

$$r = r\left(\frac{x_1}{x_2}\right),$$

or, equivalently,

$$\frac{x_1}{x_2} = f(r) .$$

The elasticity of substitution, σ, is defined as

$$\sigma = \frac{d\left(\frac{x_1}{x_2}\right)}{dr} \quad \frac{r}{\left(\frac{x_1}{x_2}\right)},$$

resulting in a number of expressions for σ. To begin with, each differential may be considered as a total differential:

$$d\,(x_1/x_2) = \frac{\partial(x_1/x_2)}{\partial x_1}\, dx_1 + \frac{\partial(x_1/x_2)}{\partial x_2}\, dx_2$$

$$= \frac{x_2 dx_1 - x_1 dx_2}{x_2^{\;2}}$$

$$dr = \frac{\partial r}{\partial x_1}\, dx_1 + \frac{\partial r}{\partial x_2}\, dx_2 .$$

$$\text{Thus: } \sigma = \frac{\dfrac{x_2 dx_1 - x_1 dx_2}{x_2^{\,2}}}{\dfrac{\partial r}{\partial x_1} dx_1 + \dfrac{\partial r}{\partial x_2} dx_2} \quad \frac{\dfrac{dx_1}{dx_2}}{\left(\dfrac{x_1}{x_2}\right)}$$

$$= \frac{\dfrac{x_2 \dfrac{dx_1}{dx_2} - x_1}{x_2^{\,2}}}{\dfrac{\partial r}{\partial x_1} + \dfrac{\partial r}{\partial x_2} \dfrac{dx_2}{dx_1}} \quad \frac{x_2}{x_1}$$

$$= \frac{r x_2 - x_1}{r\dfrac{\partial r}{\partial x_1} + \dfrac{\partial r}{\partial x_2}} \quad \frac{r}{x_1 x_2}$$

$$= \frac{-\dfrac{\partial X/\partial x_2}{\partial X/\partial x_1} x_2 - x_1}{-\dfrac{\partial X/\partial x_2}{\partial X/\partial x_1} \dfrac{\partial\left\{\dfrac{-\partial X/\partial x_2}{\partial X/\partial x_1}\right\}}{\partial x_1} + \dfrac{\partial\left\{\dfrac{-\partial X/\partial x_2}{\partial X/\partial x_1}\right\}}{\partial x_2}} \qquad \frac{-\dfrac{\partial X/\partial x_2}{\partial X/\partial x_1}}{x_1 x_2}$$

$$= \frac{\left[\dfrac{\partial X}{\partial x_2} x_2 + \dfrac{\partial X}{\partial x_1} x_1\right]\left[\dfrac{1}{\partial X/\partial x_1}\right] \left[\dfrac{\partial X/\partial x_2}{\partial X/\partial x_1}\right]}{-\dfrac{\partial X/\partial x_2}{\partial X/\partial x_1}\left[\dfrac{-\dfrac{\partial X}{\partial x_1}\dfrac{\partial^2 X}{\partial x_1 \partial x_2} + \dfrac{\partial X}{\partial x_2}\dfrac{\partial^2 X}{\partial x_1^{\,2}}}{\left(\dfrac{\partial X}{\partial x_1}\right)^2}\right] + \left[\dfrac{-\dfrac{\partial X}{\partial x_1}\dfrac{\partial^2 X}{\partial x_2^{\,2}} + \dfrac{\partial X}{\partial x_2}\dfrac{\partial^2 X}{\partial x_1 \partial x_2}}{\left(\dfrac{\partial X}{\partial x_1}\right)^2}\right]} \frac{1}{x_1 x_2}$$

$$= \frac{\left[\frac{\partial X}{\partial x_2} x_2 + \frac{\partial X}{\partial x_1} x_1\right]\left[\frac{\partial X}{\partial x_2}\right]\left[\frac{\partial X}{\partial x_1}\right]}{-\left(\frac{\partial X}{\partial x_1}\right)^2 \frac{\partial^2 X}{\partial x_2^2} - \left(\frac{\partial X}{\partial x_2}\right)^2 \frac{\partial^2 X}{\partial x_1^2} + 2\frac{\partial X}{\partial x_1}\frac{\partial X}{\partial x_2}\frac{\partial^2 X}{\partial x_1 \partial x_2}} \quad \frac{1}{x_1 x_2} .$$

The final expression for σ is again entirely in terms of the various marginals of the table. Substituting these values produces the following results:

$$\sigma = \begin{cases} 1 & \text{for the C-D} \\ \frac{1}{1+\rho} & \text{for the C. E. S.} \end{cases}$$

Hence, both forms have constant elasticities of substitution. However, in the C. E. S. case, its value depends upon one of the parameters of the function, while in the C-D case, σ is seen to be identically equal to one. This result can be used to aid in discriminating between the two forms of the production function.

The C. E. S. case can be further examined by looking at this parameter, ρ, which figures in the equation for σ. It was shown earlier that ρ is restricted, such that $-1 < \rho < 0$ or $0 < \rho$. We can now examine the limiting behavior of the C. E. S. function as ρ approaches the values $-1, 0,$ and ∞.

1. $\rho \to 0$. In this case, it can be shown that the C. E. S. function approaches the C-D form as follows:

Write the C. E. S. as

$$X = Y [\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho}]^{-(1/\rho)}$$

and rearrange, giving

$$\left(\frac{X}{Y}\right)^{-\rho} = \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} .$$

Mathematical manipulations then yield:

$$\exp\left[-\rho \ln \frac{X}{Y}\right] = \delta \exp[-\rho \ln x_1] + (1-\delta) \exp[-\rho \ln x_2]$$

and so

$$1 - \rho \ln\left(\frac{X}{Y}\right) + O(\rho^2) = 1 - \delta\rho \ln x_1 - (1-\delta) \rho \ln x_2 + O(\rho^2) .$$

Dividing by ρ and taking the limit $\rho \to 0$, we get:

$$X = Y x_1^{\delta} x_2^{(1-\delta)} ,$$

which is the C-D function with constant returns to scale. Hence, the C-D can be viewed as a particular limiting form of the C. E. S. function.

2. $\rho \to -1$. This approaches the case of perfect substitution. The function is given by

$$X = M[Ax_1 + Bx_2] .$$

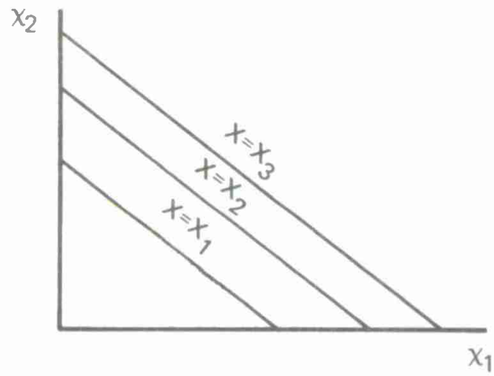
We see that

$$\frac{\partial X}{\partial x_1} = MA$$

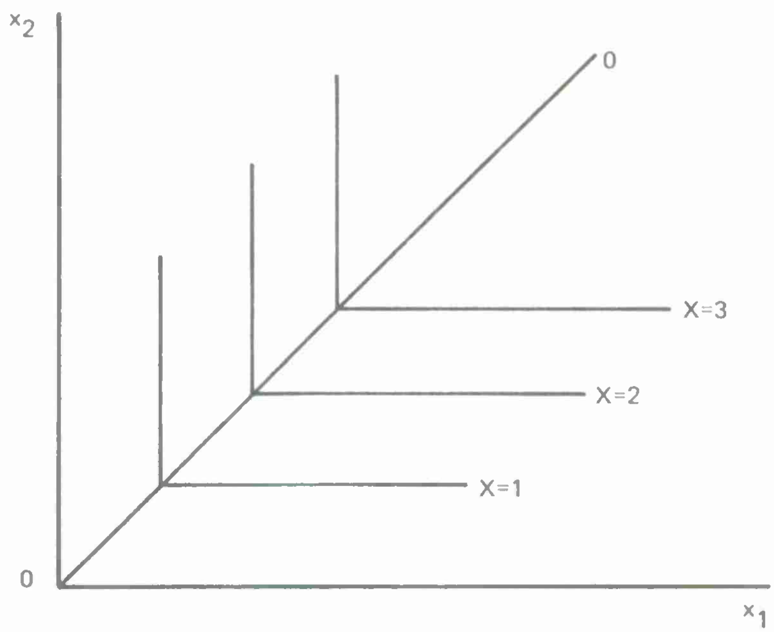
$$\frac{\partial X}{\partial x_2} = MB$$

$$\frac{\partial^2 X}{\partial x_i^2} = 0 \text{ for all } i .$$

The marginal productivities are now constant and no longer diminish, as shown in the isoquant map below.



3. $\rho \rightarrow \infty$. This is the case of zero elasticity of substitution, also known as the case of technologically fixed input proportions, shown in the isoquant map below:



Clearly, any input combination not on the ray 00' is inefficient, in that inputs are wasted. If we define a_i as the amount of input i required per unit of output, the production function can be written

$$X = \min_i \left(\frac{X_i}{a_i} \right).$$

This "process" concept is the basis for the linear programming models of optimal input and output.

This summarizes the basic theoretical concepts of a production function. While these results are presented for a simplified, two-input production function, the analysis of the more general form follows similar lines. The types of results derived here are those employed in the study. We will be primarily interested in determining the marginal products of the various inputs to the squadron; that is, we wish to determine the effect on squadron output of increases in these various inputs. In addition, we will determine the elasticity of substitution of the various inputs, which will indicate the potential for trade-offs among them, and attempt to estimate, as far as is possible, the effect on output of proportional changes in all input levels. These types of results are reported later in this volume.

ECONOMETRIC ESTIMATION OF PRODUCTION FUNCTIONS

This section discusses the problems involved in an econometric estimation of the parameters of the two-input production functions just described. The section concludes with a description of the technique used in arriving at the parameter estimates presented later.

ESTIMATION OF THE COBB-DOUGLAS PRODUCTION FUNCTION

The Cobb-Douglas (C-D) form of the production function, due to its early development and to the relative ease with which its parameters can be estimated, has been widely employed in empirical production function analyses. Writing the Cobb-Douglas function as:

$$X_t = Mx_{1t}^{\alpha} \; x_{2t}^{\beta} \; u_t \; ,$$

where the t subscript indicates a particular (time series or cross-sectional) observation and u_t is a random disturbance, a logarithmic transformation yields the equation:

$$\ln X_t = \ln M + \alpha \ln x_{1t} + \beta \ln x_{2t} + \ln u_t \; .$$

Treating $\ln u_t$ as an additive random disturbance with a zero mean, the function may be treated as a single equation that is linear in the unknown parameters $\ln M$, α and β. These parameters may then be estimated by the method of least squares or, if the further assumption is made that $\ln u_t$ is normally distributed with zero mean and constant variance σ^2, by the method of maximum likelihood. If the assumption is made instead that u_t has the normal distribution described above, then the attention of this estimating technique is shifted to the conditional median of the function, rather than to the conditional mean.

There are at least two other ways in which the simplicity of this estimating technique can be lost. If the random disturbance is postulated to be additive rather than multiplicative, or if auxiliary conditions requiring cost minimization in the competitive markets for the input factors are assumed, the estimation is no longer a simple linear problem. In the latter case, single equation least-squares estimates of the parameters have been shown to be biased and inconsistent.

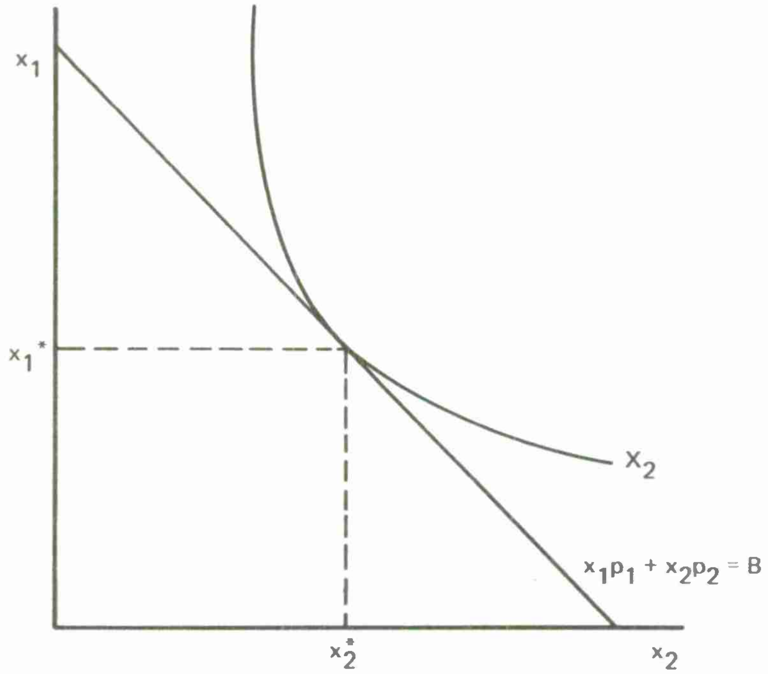
ESTIMATION OF THE CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION

Despite these difficulties, estimating the Cobb-Douglas form is far simpler than estimation of the C. E. S. form. Writing the C. E. S. function in the alternative form derived in the previous section,

$$X_t = A\left[\delta x_{1t}^{-\rho} + (1-\delta)x_{2t}^{-\rho}\right]^{-\frac{\mu}{\rho}},$$

it can be seen that, regardless of whether the random disturbance is postulated as additive or multiplicative, no logarithmic or other simple transformation can be used to yield an equation that can be estimated by standard linear regression techniques. This has necessitated the use of either non-linear techniques or auxiliary conditions in the estimation. We shall consider the latter techniques first.

In a competitive input market, the maximum output that can be achieved within a specified budget, where the price of x_1 is p_1 and the price of x_2 is p_2, is determined at the point of tangency between the budget line and the isoquant, as shown below. This result is derived in the section entitled "Theory of Cost Functions."



The marginal productivity relationship based upon output maximization is:

$$\frac{P_1}{P_2} = \frac{\partial X_t}{\partial x_{1t}} \bigg/ \frac{\partial X_t}{\partial x_{2t}}$$

$$= \frac{\delta}{1-\delta}\left(\frac{x_{1t}}{x_{2t}}\right)^{-(\rho+1)}$$

The first step in estimating the parameters of the C. E. S. function is to transform the above equation logarithmically into:

$$\log\left(\frac{p_{1t}}{p_{2t}}\right) = \log\left(\frac{\delta}{1-\delta}\right) - (\rho+1)\log\left(\frac{x_{1t}}{x_{2t}}\right)$$

The regression of $\log\left(\frac{p_{1t}}{p_{2t}}\right)$ on $\log\left(\frac{x_{1t}}{x_{2t}}\right)$ will provide estimates of δ and ρ.

The second step is to transform the equation for the C. E. S. function logarithmically into:

$$\log X_t = \log A - \frac{\mu}{\hat{\rho}} \log \left[\hat{\delta} x_{1t}^{-\hat{\rho}} + (1-\hat{\delta}) x_{2t}^{-\hat{\rho}} \right]$$

We can now estimate A and μ from the regression of $\log X_t$ on $\log \left[\hat{\delta} x_{1t}^{-\hat{\rho}} + (1-\hat{\delta}) X_{2t}^{-\hat{\rho}} \right]$.

As shown in the above procedure, the assumption of competitive factor markets allows the use of standard linear regression to estimate the parameters of the production function. However, even if competitive market auxiliary conditions are assumed, the estimates depend on the specification of the variables that are to be treated as exogenous.

If we do not rely upon market condition information, we must turn to non-linear methods to estimate the parameters. The following two procedures use iterative techniques for minimizing the sum of the squared errors between the actual and estimated output, given by:

$$S = \sum_t \left(X_t - A \left[\delta x_{1t}^{-\rho} + (1-\delta) x_{2t}^{-\rho} \right]^{-\frac{\mu}{\rho}} \right)^2 .$$

For the first procedure we select initial values of the parameters A_o, δ_o, ρ_o, μ_o, and expand the production function in the following form:

$$X_{ot} = A_o \left[\delta_o x_{1t}^{-\rho_o} + (1-\delta_o) x_{2t}^{-\rho_o} \right]^{-\mu_o/\rho_o}$$

$$+ \left(\frac{\partial X_t}{\partial A} \right)_o (A-A_o) + \left(\frac{\partial X_t}{\partial \delta} \right)_o (\delta-\delta_o)$$

$$+ \left(\frac{\partial X_t}{\partial \rho} \right)_o (\rho - \rho_o) + \left(\frac{\partial X_t}{\partial \mu} \right)_o (\mu-\mu_o)$$

The Taylor series expansion of degree one provides a linear approximation of the function.

The next step is to find estimates of A, δ, ρ, and μ which minimize the expression:

$$S_o = \sum_t (X_t - X_{ot})^2$$

The estimates A_1, δ_1, ρ_1, and μ_1 are then used to form the linear equations as above. We then proceed to minimize:

$$S_o = \sum_t (X_t - X_{1t})^2 .$$

This will produce a second set of estimates A_2, δ_2, ρ_2, and μ_2. One can continue this procedure until the estimates converge to a particular set of values.

The other method is to form the system of simultaneous non-linear equations by setting the partial derivatives of S, given above, equal to zero:

$$\frac{\partial S}{\partial A} = 0, \quad \frac{\partial S}{\partial \delta} = 0, \quad \frac{\partial S}{\partial \rho} = 0, \quad \frac{\partial S}{\partial \mu} = 0 \ .$$

Each of these equations can be expanded by use of a Taylor's series around initial values of the parameters A, δ, ρ, and μ. This will provide a system of linear equations in the unknown parameters.

Each equation will be of the following form:

$$\frac{\partial S}{\partial A} = \left(\frac{\partial S}{\partial A}\right)_o + \left(\frac{\partial^2 S}{\partial A^2}\right)_o (A - A_o)$$

$$+ \left(\frac{\partial^2 S}{\partial A\, \partial \delta}\right)_o (\delta - \delta_o) + \left(\frac{\partial^2 S}{\partial A\, \partial \rho}\right)_o (\rho - \rho_o)$$

$$+ \left(\frac{\partial^2 S}{\partial A\, \partial \mu}\right)_o (\mu - u_o)$$

The system of equations can be solved for A_1, δ_1, ρ_1 and μ_1. We can then proceed iteratively until the estimates converge to a set of values.

The derivation of maximum likelihood estimates would involve the same technique. We would set up the likelihood function based upon a specified distribution of the residuals and set the partial derivatives equal to zero. A system of nonlinear equations could again be linearized by expanding the functions in a Taylor series around initial values of the parameters. A Bayesian procedure using maximum likelihood techniques might be of value, if strong prior information is available.

A suggested iterative procedure begins by selecting initial values of the two parameters δ and ρ. The regression of $\log X_t$ on $\log [\delta_o x_{1t}^{-\rho_o} + (1-\delta_o) x_{2t}^{-\rho_o}]$ will provide estimates of the remaining parameters A and μ. The regression equation is of the following form:

$$\log X_t = \log A - \frac{\mu}{\rho_o} \log [\delta_o x_{1t}^{-\rho_o} + (1-\delta_o) x_{2t}^{-\rho_o}] .$$

Using the estimates A_o and μ_o and the initial values selected for the parameters δ and ρ, the quantity Y_{ot} can be computed:

$$Y_{ot} = \left[\left(\frac{X_t}{A_o} \right)^{-\rho_o/\mu_o} - (1-\delta_o) x_{2t}^{-\rho_o} \right] = \delta x_{1t}^{-\rho} .$$

New estimates α_1 and δ_1 can be obtained from the regression

$$\log Y_{ot} = \log \delta - \rho \log x_{1t} .$$

Using the new estimates δ_1 and ρ_1 the procedure can be repeated until the procedure converges to a particular set of estimates.

The C. E. S. function can be approximated by the first and second-order terms in a Taylor series expansion. The function, which includes the Cobb-Douglas, has the following form:

$$\log X_t = a_o + a_1 \log x_{1t} + a_2 \log x_{2t} + a_3 \left(\log \left(\frac{x_{1t}}{x_{2t}} \right) \right)^2$$

where

$$\log A = a_o$$

$$\delta = a_1 / (a_1 + a_2)$$

$$\rho = - 2a_3 (a_1 + a_2) / a_1 a_2$$

$$\mu = a_1 + a_2 .$$

This procedure is advantageous since one can estimate the parameters of the function by running a simple linear regression on the variables as shown in the equation above.

The estimation procedure employed in this study is of the general class of non-linear, iterative techniques described on the preceding page. The alternative class of estimation techniques, those employing auxiliary market information, was felt to be inappropriate, considering the nature of the markets in which the Navy procures the inputs of labor, aircraft, and spare parts. The procedure guarantees convergence to a local extremum, but we cannot be certain that we have found the global minimum of the squared error loss function. A procedure to determine the goodness of fit from the estimated parameters has been incorporated into the technique to insure against convergence to a local maximum.

The form of the production function used in the estimates presented in the following section can be written as:

$$Y_t = A \left[\alpha_1 x_{1t}^{-\rho_1} + \alpha_2 x_{2t}^{-\rho_2} + \alpha_3 x_{3t}^{-\rho_3}\right]^{-\frac{1}{\rho}},$$

where Y represents the actual ready hour production, x_{1t}, x_{2t}, and x_{3t} represent the usage of aircraft, maintenance labor, and spare parts, respectively, and the t subscript denotes the particular observation. The construction of these variables is described in detail in appendix B. There are 2 primary differences between this form of the equation and the forms described in the earlier sections of this paper. The first is the addition of a third input variable; although this leads to greater mathematical complexity, the extension of the theory to the 3-variable case is straightforward. The second difference is in the subscripting of the exponents attached to the input variables in the function. This modification was made so that the elasticity of substitution between each pair of inputs could be separately identified. Since there is no a priori reason to believe that the 3 combinations of planes and maintenance labor, planes and spare parts, and spare parts and maintenance labor should exhibit the same elasticity of substitution, this complexity was introduced.

The procedure employed involves the minimization of the squared error loss function

$$S = \sum_t \left\{Y_t - A\left[\alpha_1 x_{1t}^{-\rho_1} + \alpha_2 x_{2t}^{-\rho_2} + \alpha_3 x_{3t}^{-\rho_3}\right]^{-\frac{1}{\rho}}\right\}^2$$

with respect to the unknown parameters A_1, α_1, α_2, α_3, ρ_1, ρ_2, ρ_3, and ρ.

In order to do this, a system of non-linear equations is formed in the unknown parameters:

$$\frac{\partial S}{\partial A} = 0;$$

$$\frac{\partial S}{\partial \alpha_i} = 0 \qquad i = 1, 2, 3;$$

$$\frac{\partial S}{\partial \rho_i} = 0 \qquad i = 1, 2, 3;$$

$$\frac{\partial S}{\partial \rho} = 0.$$

Each of these equations can be expanded about assumed initial values of the parameters as follows:

$$\begin{aligned}
\frac{\partial S}{\partial A} &= \left(\frac{\partial S}{\partial A}\right)_o + \left(\frac{\partial^2 S}{\partial A^2}\right)_o \left(A - A^o\right) \\
&+ \left(\frac{\partial^2 A}{\partial A\, \partial \alpha_1}\right)_o \left(\alpha_1 - \alpha_1{}^o\right) + \left(\frac{\partial^2 S}{\partial A\, \partial \alpha_2}\right)_o \left(\alpha_2 - \alpha_2{}^o\right) \\
&+ \left(\frac{\partial^2 S}{\partial A\, \partial \alpha_3}\right)_o \left(\alpha_3 - \alpha_3{}^o\right) + \left(\frac{\partial^2 S}{\partial A\, \partial \rho_1}\right)_o \left(\rho_1 - \rho_1{}^o\right) \\
&+ \left(\frac{\partial^2 S}{\partial A\, \partial \rho_2}\right)_o \left(\rho_2 - \rho_2{}^o\right) + \left(\frac{\partial^2 S}{\partial A\, \partial \rho_3}\right)_o \left(\rho_3 - \rho_3{}^o\right) \\
&+ \left(\frac{\partial^2 S}{\partial A\, \partial \rho}\right)_o \left(\rho - \rho^o\right) \\
&= 0,
\end{aligned}$$

where the 0 indicates evaluation at the assumed initial values.

Similar expansions for $\frac{\partial S}{\partial \alpha_i}$, $\frac{\partial S}{\partial \rho_i}$, and $\frac{\partial S}{\partial \rho}$ about the initial values A^o, α_i^o, ρ_i^o, and ρ^o may then be solved for new parameter estimates, and these new estimates are used iteratively until a final set of parameter estimates is reached.

The final question to be discussed in this section involves the problem of choosing between the C-D and the C.E.S. forms of the production function. Since we have already seen earlier that the C-D form is a special limiting case of the C.E.S. function, the question arises as to why the C-D function should ever be employed at all. The advantage of using the C.E.S. form of the production function is that elasticities of substitution different from one can be estimated. If, however, the estimated elasticities are statistically indistinguishable from this value, there is no reason to apply the complex C.E.S. form rather than the simpler Cobb-Douglas function. Recent studies have suggested that in many instances the Cobb-Douglas form is adequate for analysis, as well as simpler to apply. In the sections to follow, we report on results based upon the C.E.S. form of the production function. In future studies, however, the simpler form might be employed at not too great a cost. The possibility of further stratifying the input categories (e.g., several categories of maintenance labor, spare parts, etc.) also suggests that a simpler algebraic form of the production function may be more computationally desirable.

THE THEORY OF COST FUNCTIONS

The purpose of this section is to discuss the concept of a cost function. We will use, as earlier, a simple 2-input production function. The cost function results from choosing the minimum cost combination of inputs leading to the production of a given amount of output, or, alternatively, from choosing the combination of inputs that leads to the greatest level of output while satisfying a budget constraint.

THE COBB-DOUGLAS PRODUCTION FUNCTION

In order to derive a cost function, we must use a budget constraint. We assume that a firm has a fixed budget, C, which it wishes to allocate among the inputs in such a way that the greatest potential output for that budget level is achieved:

$$p_1 x_1 + p_2 x_2 \leq C \, ,$$

where p_1 and p_2 represent the unit costs of the two inputs. Initially, we will also assume that the firm has a production function of the Cobb-Douglas form.

Since we know that the marginal product of any input is always positive, output cannot be maximized unless the budget constraint is satisfied with equality. Hence, we can rewrite our production function as

$$\begin{aligned} X &= M \; x_1^{\alpha} x_2^{\beta} \\ &= M \; x_1^{\alpha} \left(\frac{C - p_1 x_1}{p_2} \right)^{\beta} , \end{aligned}$$

using the fact that $x_2 = \dfrac{C - p_1 x_1}{p_2}$.

Then, for an output maximum, we have the necessary and sufficient conditions

$$\frac{dX}{dx_1} = M \; x_1^{\alpha-1} \left(\frac{C-p_1x_1}{p_2}\right)^{\beta-1} \left(\alpha \; \frac{C-p_1x_1}{p_2} - \beta \; \frac{p_1}{p_2} \; x_1\right) = 0 \; ,$$

$$\frac{d^2X}{dx_1^2} = - \; (\alpha+\beta) \;\; M \; \frac{p_1}{p_2} \; x_1^{\alpha-1} \left(\frac{C-p_1x_1}{p_2}\right)^{\beta-1} < 0 \; .$$

Using the various restrictions on the signs and magnitudes of the variables and parameters, we see that the latter condition is always satisfied. Furthermore, we see that $\frac{dX}{dx_1} = 0$ if and only if $\left(\alpha \frac{C-p_1x_1}{p_2} - \beta \; \frac{p_1}{p_2} \; x_1\right) = 0$.

We may then solve for x_1 , giving

$$x_1 = \frac{\alpha}{\alpha+\beta} \quad \frac{C}{p_1} \; ,$$

and for x_2 , using the budget constraint, giving

$$x_2 = \frac{\beta}{\alpha+\beta} \quad \frac{C}{p_2} \; .$$

These two equations may be viewed as input demand functions. Inserted into the production function gives

$$X = \frac{M}{(\alpha+\beta)^{\alpha+\beta}} \quad \left(\frac{\alpha}{p_1}\right)^{\alpha} \left(\frac{\beta}{p_2}\right)^{\beta} \quad C^{\alpha+\beta} \; .$$

This equation describes the level of output, X , as a function of the fixed cost budget, C , when inputs are applied in the most efficient proportion, namely

$$\frac{x_1}{x_2} = \frac{\alpha}{\beta} \quad \frac{p_2}{p_1} \; .$$

Alternatively, the minimum cost of attaining a given output level may be written as

$$C = (\alpha+\beta)\left[\frac{1}{M}\left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{\beta}\right)^{\beta}\right]^{\frac{1}{\alpha+\beta}} X^{\frac{1}{\alpha+\beta}} .$$

Unit or average cost is easily obtained, giving

$$\frac{C}{X} = (\alpha+\beta)\left[\frac{1}{M}\left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{\beta}\right)^{\beta}\right]^{\frac{1}{\alpha+\beta}} X^{\frac{1}{\alpha+\beta}-1} .$$

Finally, marginal cost is given by

$$\frac{dC}{dX} = \left[\frac{1}{M}\left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{\beta}\right)^{\beta}\right]^{\frac{1}{\alpha+\beta}} X^{\frac{1}{\alpha+\beta}-1} .$$

The elasticity of cost with respect to output, ξ, is given by the ratio of marginal to unit cost, or

$$\xi = \frac{dC}{dX}\;\frac{X}{C} = \frac{1}{\alpha+\beta} .$$

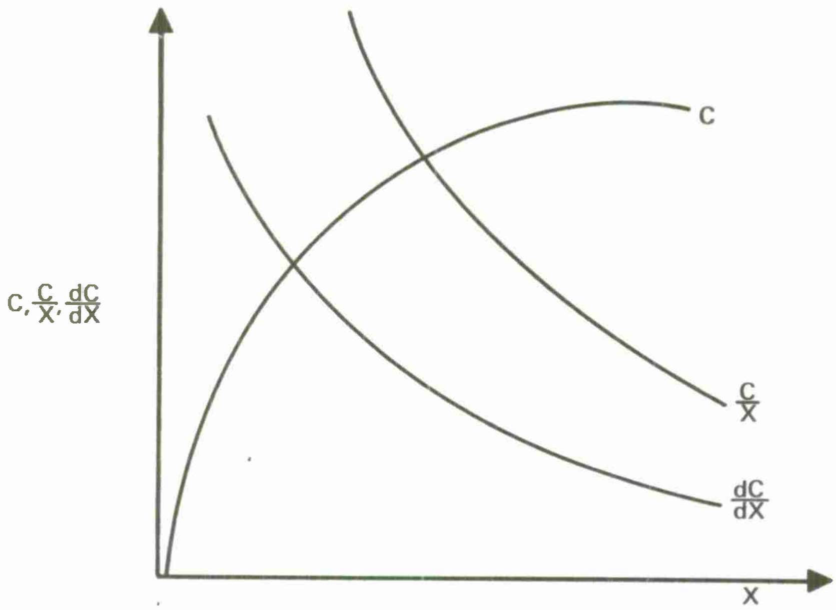
The figures below demonstrate the distinction among increasing, constant, and decreasing returns to scale.

These results indicate the importance of the estimated degree of homogeneity of the production function, discussed earlier. If the estimated degree of homogeneity is in the range of decreasing returns to scale (i. e., $\alpha+\beta < 1$), then the marginal cost of increasing output is greater than the average cost per unit of output.

The relationship between the cost function and the auxiliary market conditions, discussed in regard to estimation of the C. E. S. function, can be seen more clearly by an alternative method of derivation. The cost function results from the solution to the constrained optimization problem

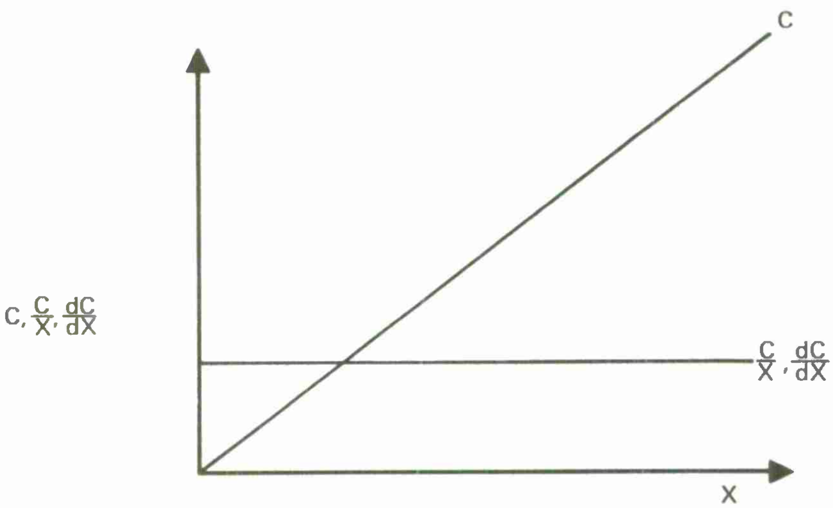
$$\text{maximize} \quad X = M\, x_1^{\alpha} x_2^{\beta}$$

$$\text{subject to} \quad p_1x_1 + p_2x_2 = C$$



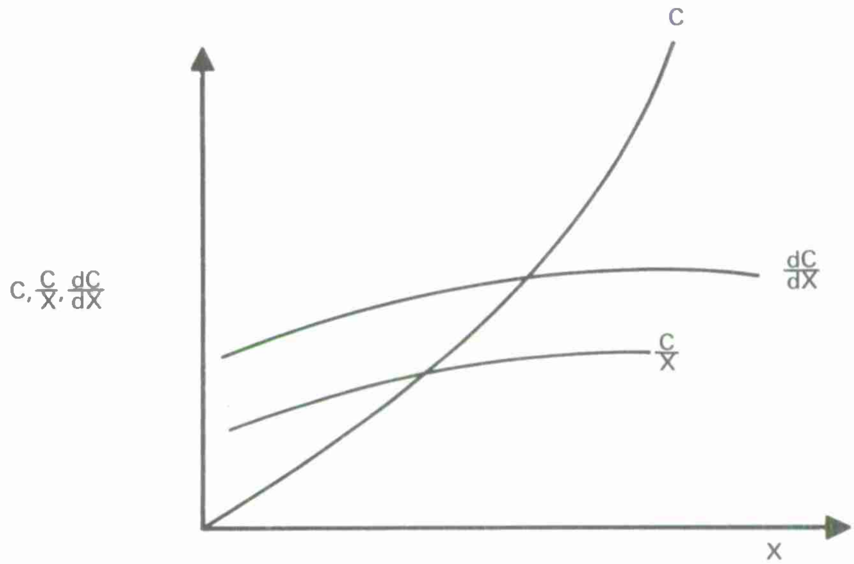
CASE I: $\alpha + \beta > 1$ (increasing returns to scale)

Note that $\frac{C}{X} > \frac{dC}{dX}$



CASE II: $\alpha + \beta = 1$ (constant returns to scale)

Note that $\frac{C}{X} = \frac{dC}{dX}$



CASE III: $\alpha + \beta < 1$ (decreasing returns to scale)

Note that $\frac{C}{X} < \frac{dC}{dX}$

and the parameterization of the budget level C. This optimization can be solved by using the method of LaGrangian multipliers, forming the function

$$L = Mx_1^{\alpha} x_2^{\beta} + \lambda [p_1x_1 + p_2x_2 - C] .$$

We may find the maximum by solving the set of equations

$$\frac{\partial L}{\partial x_1} = \frac{\alpha X}{x_1} + \lambda p_1 = 0 ,$$

$$\frac{\partial L}{\partial x_2} = \frac{\beta X}{x_2} + \lambda p_2 = 0 ,$$

$$\frac{\partial L}{\partial \lambda} = p_1x_1 + p_2x_2 - C = 0 .$$

For the cases of constant or decreasing returns to scale, second-order conditions are seen to be satisfied by noting that the principle minors of the bordered Hessian determinant

$$\begin{bmatrix} \frac{\alpha(\alpha-1)X}{x_1^2} & \frac{\alpha\beta X}{x_1x_2} & p_1 \\ \frac{\alpha\beta X}{x_1x_2} & \frac{\beta(\beta-1)X}{x_2^2} & p_2 \\ p_1 & p_2 & 0 \end{bmatrix}$$

alternate in sign. The efficient proportion in which to apply the inputs is then derived from the solution of the first two equations:

$$\frac{x_1}{x_2} = \frac{\alpha}{\beta}\,\frac{p_2}{p_1},$$

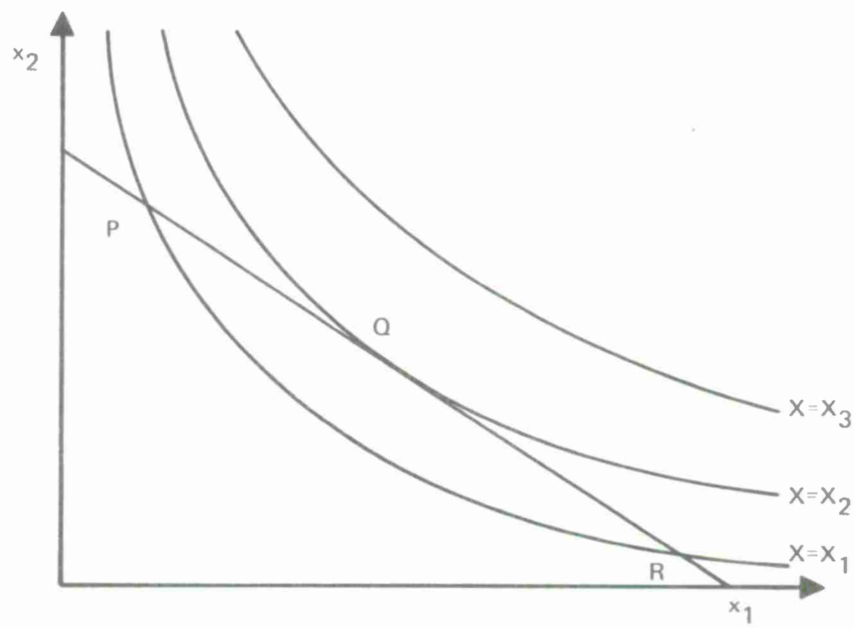
and the cost function can be derived by using either of the first 2 equations and the third. Rewriting the above equation as

$$\frac{\dfrac{\alpha X}{x_1}}{\dfrac{\beta X}{x_2}} = \frac{p_1}{p_2}$$

we see that the term on the left is just $\dfrac{\partial X/\partial x_1}{\partial X/\partial x_2}$, or the negative of the slope of the isoquant, and that the term on the right is the negative of the slope of the budget constraint, which can be rewritten as

$$x_2 = \frac{C}{p_2} - \frac{p_1}{p_2}\,x_1 .$$

Multiplying each of these terms by minus one gives the familiar condition that at the optimal point the slope of the isoquant must be the same as the slope of the budget constraint. This may be illustrated as the tangency of the isoquant and the budget constraint, as shown below.



Clearly, the point Q represents the optimal choice of inputs; any other combination of inputs satisfying the budget constraint, such as P and R, are on a lower isoquant and hence lead to a lower level of output. The use of LaGrangian multipliers is the technique used in arriving at the cost functions in this study.

THE CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION

For the case of the C. E. S. production function, the analysis follows the same path as in the C-D case. Similar results are obtained. A total cost function, representing the minimum cost of producing a given level of output, is given by

$$C = QX ,$$

where

$$Q = \frac{1}{M}\left[(Ap_1{}^{\rho})^{\frac{1}{1+\rho}} + (Bp_2{}^{\rho})^{\frac{1}{1+\rho}} \right]^{\frac{1+\rho}{\rho}} ,$$

and the most efficient proportion in which to apply the inputs is given by

$$\frac{x_1}{x_2} = \left(\frac{A}{B} \quad \frac{p_2}{p_1} \right)^{\frac{1}{1+\rho}} .$$

Since we have a linearly homogeneous production function, we expect total cost to be in direct proportion to output.

Finally, it is easily seen that

$$\frac{C}{X} = \frac{dC}{dX} = Q$$

and

$$\xi = \frac{dC}{dX} \frac{X}{C} = 1 .$$

That is, marginal and unit cost are identical, and the elasticity of cost with respect to output is identically equal to one. The analysis leading to the cost function employed in this study is found in the next to last section: Results of Cost Function Analysis. Again, the concepts are similar to those presented above.

RESULTS FROM THE PRODUCTION FUNCTION ANALYSES

This section presents the empirical results from the production function analysis of each of the 5 aircraft considered in this study. The estimated function is presented for each aircraft under several separate breakdowns: Atlantic-based squadrons, Pacific-based squadrons, and, when appropriate, training squadrons, along with estimates based upon the full cross-section of observations. The first 3 breakdowns of the data were analyzed to reflect differences in the operating environments facing Atlantic, Pacific, and training squadrons. To the extent that these operating environments can be expected to continue as they have in the past, these separate estimates may be of value in analyzing potential changes in these areas.

While a principle purpose of deriving these production function estimates is to provide an input to the cost function analysis described in the following sections, a number of useful results can be derived from the production function analysis alone. The production function enables us to estimate the level of ready hours that will be produced with various combinations of inputs. For each type/model/series of aircraft and for each location/deployment combination considered, this type of result is demonstrated in a set of tables and graphs. These tables and graphs represent only one set of possible input combinations that may be of interest to the planner; the construction of alternative tables and graphs for different combinations can be easily accomplished with the use of computer routines developed in this study, as discussed in appendix B. A second result of this stage of the analysis is the estimated returns to scale for such type/model/series studied. These latter results are derived basically from the Cobb-Douglas form of the production function, although they have been verified by use of the C. E. S. form and the approximation $\rho_1 = \rho_2 = \rho_3 = \rho$. Finally, estimated elasticities of substitution, derived from the C.E.S. form of the production function, are presented.

These results are presented separately for each of the 5 aircraft studied; a detailed description of how the various tables and charts may be used is contained in the discussion of the A-7B. It is important to note in examining these tables and graphs that the maintenance man-hour figures relate only to the portion of total maintenance time addressed to the removal, repair, and installation of spare parts, and not to the other portions of total maintenance time employed in other activities (flight preparation, etc.). For example, the planning factors for the various aircraft types imply that a total of 12,768 organizational and intermediate level maintenance man-hours should be provided to a TA-4F squadron of 10 planes, to be used for all types of maintenance activities. Similar figures for squadrons of 12 F-4B aircraft and of 17 CH-53 helicopters are 24,264 and 19,543 maintenance man-hours, respectively. For squadrons of 14 A-7B aircraft and of 10 S-2E aircraft, corresponding total maintenance man-hour figures are 35,098 and 16,322, respectively. Thus the maintenance man-hours totals reported in the following tables do not reflect total maintenance support levels for the squadrons, but only the portion directly applied to removal, repair, and installation of parts.

RESULTS FOR THE A-7B

The results of the production function analysis for the A-7B attack aircraft are presented in the following tables and graphs. For each of the 4 breakdowns studied (all squadrons, Atlantic squadrons, Pacific squadrons, and training squadrons), the following data is displayed:

- The number of observations used in the estimation,

- Average values of the variables used in the estimation,

- The form of the estimated production function,

- The R^2 statistic, giving the fraction of the total variance in observed ready hours explained by the production function model,

- The pairwise elasticities of substitution between the inputs, calculated at the average level of inputs,

- The estimated returns to scale parameter,

- Tables of the ready-hour production resulting from various levels of each of 3 inputs, holding the others constant at representative values,

- Graphs of the level of ready hours resulting from various levels of the 3 inputs.

These tables and graphs may be used in the following manner. Suppose one is interested in the level of readiness that can be achieved with various levels of spare parts support in an Atlantic squadron of 14 A-7B aircraft when 5000 man-hours of maintenance are available. The ready-hour production for various levels of spare parts usage for Atlantic A-7B squadrons is displayed in figure 1. For example, the use of $300,000 worth of spares can be expected to lead to the production of approximately 6414 ready hours. Increasing this usage by 1/6 (or by $50,000 is seen to lead to an additional 97 ready hours. These results are also displayed graphically. Note that the estimated production functions display all of the intuitively desired properties discussed in the first section.

The results here make no mention of the optimal combination of resources: this aspect of the problem is addressed in later sections. The results of this section are basically descriptive in nature; they display the expected output that will result from various combinations of inputs. This can be of significant value, especially in the light of current fiscal guidance and budgeting procedures, which restrict the number of categories among which tradeoffs can be made.

A number of interesting facts, taken from the general tables and graphs presenting the results for the A-7B, are summarized below.

1. The production function model explains a significant amount of the variance in observed ready-hour production.

2. All 3 input variables appear to have influenced the level of ready-hour production. The number of aircraft is, of course, the most important of the 3 inputs. Spare parts support appears to have been of greatest importance to Atlantic-based squadrons.

3. Statistically, the 3 inputs all have co-efficients significantly different from zero in almost all cases. This result was taken from the estimation of the Cobb-Douglas form of the production function, using a standard one-tailed t-test. The significance level of the various co-efficients is shown below:

	Planes (percent)	Man-hours (percent)	Spares (percent)
All squadrons	99	-	95
Atlantic squadron	99	-	99
Pacific squadrons	99	75	75
Training squadrons	99	99	-

4. Average ready-hour production was only about half of the possible 720 hours per aircraft per month and much lower for training squadrons:

	All	Atlantic	Pacific	Training
Average ready hours per aircraft	349	419	411	272

5. All pairs of inputs exhibited high degrees of substitutability.

6. The returns to scale parameter was always estimated as being below one, indicating that the marginal cost of an additional ready hour is higher than the average cost. For example, increasing all inputs to Pacific squadrons by 10 percent leads to only a 5.8 percent increase in ready hours. For training squadrons, this effect is even more noticeable.

TABLE 2

A-7B SQUADRONS

ALL

Number of observations: 109

Average value of data:

Ready-hours = 6452.6
Planes = 18.5
Maintenance man-hours = 5964.2
Spare parts = $200,232

Estimated production function

$$RH = 512.99\left[0.80\,P^{-0.07} + 0.05\,M^{-0.12} + 0.13\,S^{-0.02}\right]^{-10.16}$$

$R^2 = 0.566$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.895$
$\sigma_{PS} = 0.978$
$\sigma_{MS} = 0.936$

Estimated returns to scale parameter = 0.61

ATLANTIC

Number of observations: 29

Average value of data:

Ready-hours = 5662.3
Planes = 13.5
Maintenance man-hours = 4121.6
Spare parts = $191,999

Estimated production function

$$RH = 172.87\left[0.89\,P^{-0.16} + 0.005\,M^{-0.005} + 0.24\,S^{-0.06}\right]^{-10.16}$$

$R^2 = 0.368$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.995$
$\sigma_{PS} = 0.938$
$\sigma_{MS} = 0.995$

Estimated returns to scale parameter = 0.61

PACIFIC

Number of observations: 49

Average value of data:

Ready-hours = 5712.8
Planes = 13.9
Maintenance man-hours = 4199.5
Spare parts = $176,370

Estimated production function

$$RH = 188.20\left[0.88\,P^{-0.17} + 0.13\,M^{-0.04} + 0.07\,S^{-0.03}\right]^{-9.66}$$

$R^2 = 0.321$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.957$
$\sigma_{PS} = 0.969$
$\sigma_{MS} = 0.968$

Estimated returns to scale parameter = 0.58

TRAINING

Number of observations: 31

Average value of data:

Ready-hours = 8475.9
Planes = 31.2
Maintenance man-hours = 10,747.9
Spare parts = $248,236

Estimated production function

$$RH = 108.5\left[0.60\,P^{-0.08} + 0.54\,M^{-0.14} + 0.05\,S^{-0.01}\right]^{-10.00}$$

$R^2 = 0.719$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.894$
$\sigma_{PS} = 0.989$
$\sigma_{MS} = 0.987$

Estimated returns to scale parameter = 0.50

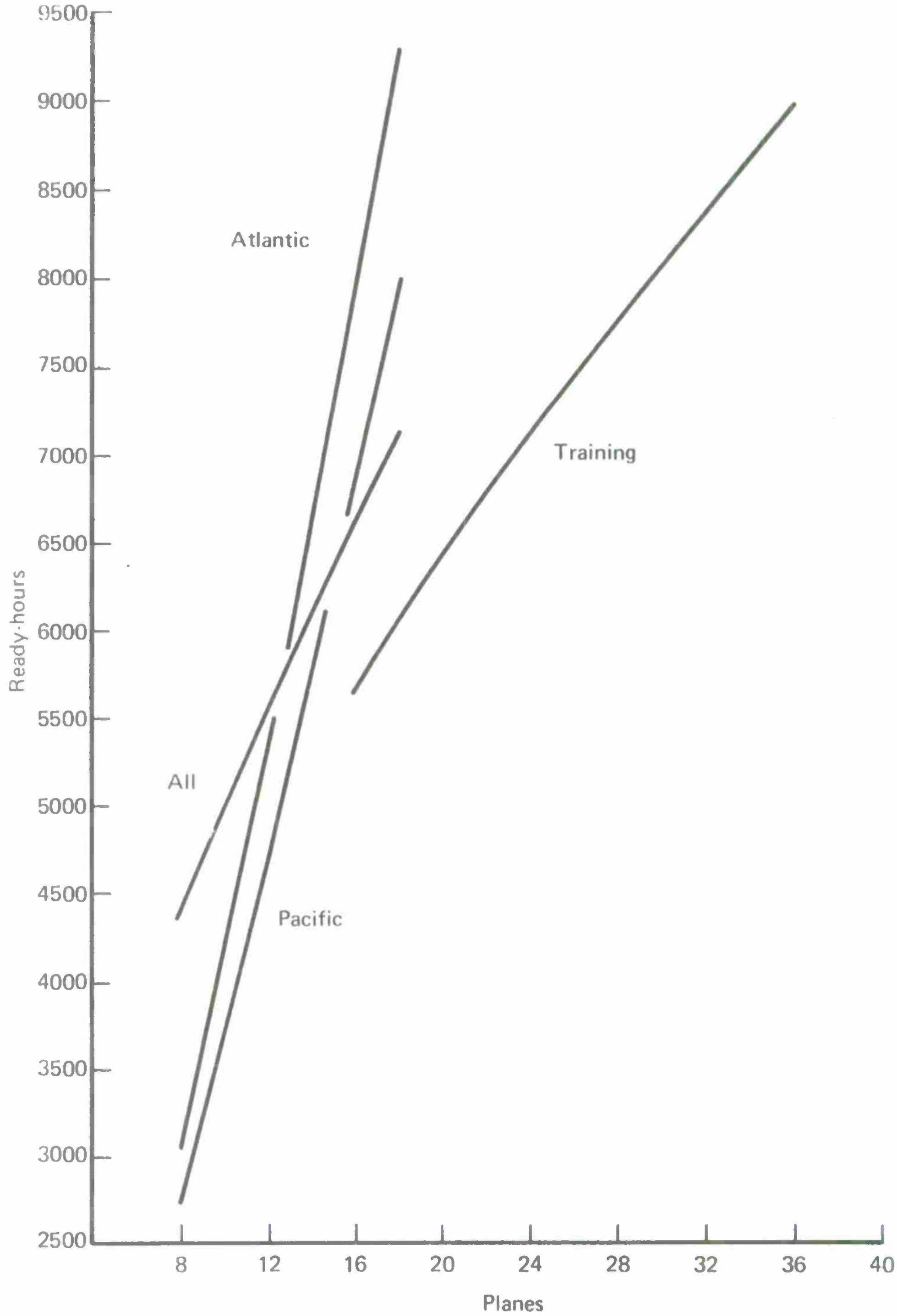


FIG. 1A: A-7B, PLANES

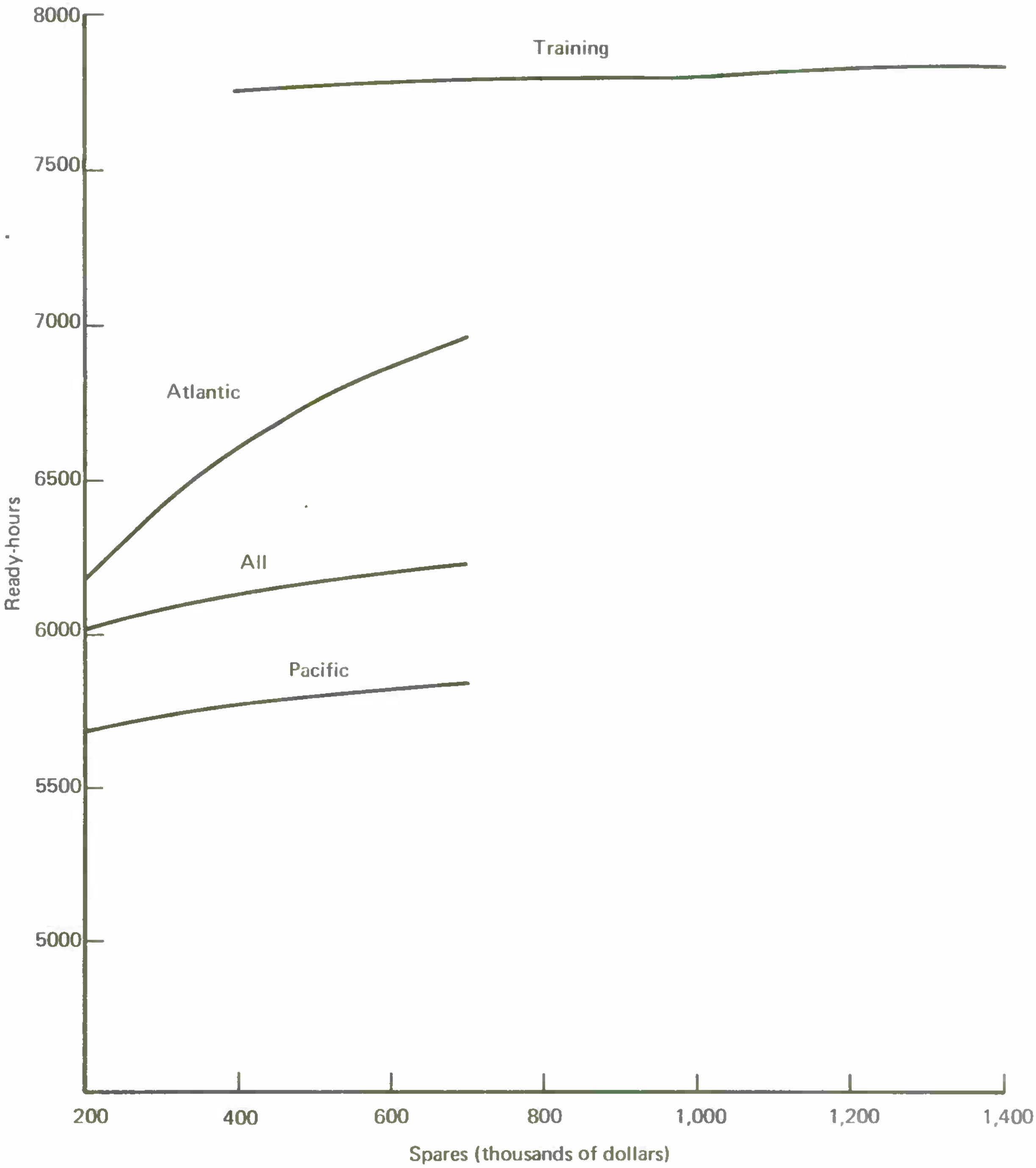


FIG. 1B: A-7B, SPARES

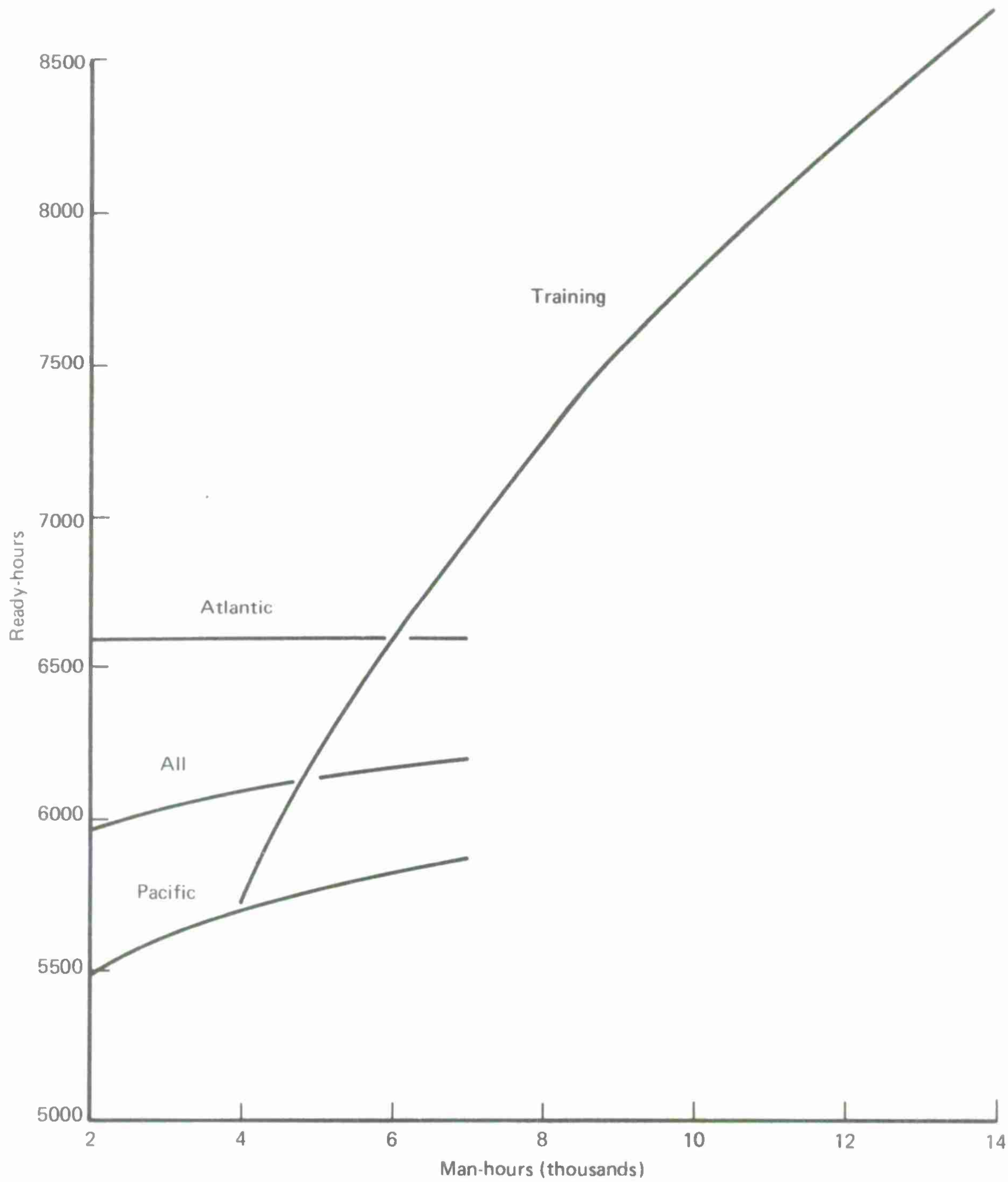


FIG. 1C: A-7B, MAINTENANCE MAN-HOURS

TABLE 3

A-7B SQUADRONS

A		B		C	
$RH = f(P, \bar{M}, \bar{S})$		$RH = f(\bar{P}, \bar{M}, S)$		$RH = f(\bar{P}, M, \bar{S})$	
Planes	Ready-hours	Spares ($)	Ready-hours	Man-hours	Ready-hours
ALL					
8	4366.9	200,000	6019.1	2,000	5965.6
9	4690.7	250,000	6054.6	2,500	6006.7
10	5000.3	300,000	6083.6	3,000	6039.8
11	5297.5	350,000	6108.2	3,500	6067.3
12	5584.1	400,000	6129.5	4,000	6090.8
13	5861.0	450,000	6148.3	4,500	6111.3
14	6129.5	500,000	6165.2	5,000	6129.5
15	6390.3	550,000	6180.4	5,500	6145.8
16	6640.0	600,000	6194.4	6,000	6160.6
17	6891.4	650,000	6207.2	6,500	6174.0
18	7133.0	700,000	6219.1	7,000	6186.4
ATLANTIC					
8	3068.0	200,000	6161.6	2,000	6593.9
9	3607.6	250,000	6299.9	2,500	6594.5
10	4168.5	300,000	6413.9	3,000	6594.9
11	4749.1	350,000	6511.2	3,500	6595.2
12	5348.0	400,000	6596.1	4,000	6595.5
13	5964.0	450,000	6671.3	4,500	6595.8
14	6596.0	500,000	6739.0	5,000	6596.1
15	7243.2	550,000	6800.4	5,500	6596.3
16	7904.8	600,000	6856.8	6,000	6596.5
17	8580.0	650,000	6908.8	6,500	6596.7
18	9268.2	700,000	6957.2	7,000	6596.8
PACIFIC					
8	2741.8	200,000	5685.5	2,000	5497.2
9	3209.5	250,000	5710.9	2,500	5561.8
10	3693.3	300,000	5731.6	3,000	5614.8
11	4191.7	350,000	5749.1	3,500	5659.8
12	4703.7	400,000	5764.2	4,000	5698.8
13	5228.1	450,000	5777.5	4,500	5733.3
14	5764.2	500,000	5789.4	5,000	5764.2
15	6311.2	550,000	5800.2	5,500	5792.2
16	6868.4	600,000	5810.0	6,000	5817.8
17	7435.3	650,000	5819.0	6,500	5841.4
18	8011.4	700,000	5827.4	7,000	5863.3
TRAINING					
16	5667.2	400,000	7750.1	4,000	5725.3
18	6061.1	500,000	7761.8	5,000	6187.7
20	6435.7	600,000	7771.3	6,000	6584.1
22	6793.6	700,000	7779.3	7,000	6932.6
24	7136.9	800,000	7786.3	8,000	7244.4
26	7467.3	900,000	7792.4	9,000	7527.2
28	7786.3	1,000,000	7797.9	10,000	7786.3
30	8095.0	1,100,000	7802.9	11,000	8025.7
32	8394.3	1,200,000	7807.4	12,000	8248.4
34	8685.1	1,300,000	7811.6	13,000	8456.8
36	8968.1	1,400,000	7815.4	14,000	8652.8

RESULTS FOR THE CH-53

The results of the production function analysis for the CH-53 helicopter are summarized in the following tables and graphs.

A number of general conclusions and results for the CH-53 are summarized below:

1. A large part of the observed variation in actual ready-hour production is explained by the model. With the exception of training squadrons, approximately 2/3 of the total variation is accounted for by the model.

2. In terms of the magnitudes of the effects of changes in the levels of the various inputs, the level of maintenance appears to have a smaller effect than do the levels of planes and spare parts.

3. Based upon the one-tailed t-test described earlier, the co-efficients of the input variables of planes and spare parts in the Cobb-Douglas estimation were, in general, statistically different from zero: the co-efficient of the man-hour variable was not.

The levels of significance are shown below:

	Planes (percent)	Man-hours	Spare parts (percent)
All squadrons	99	-	99
Atlantic squadrons	95	-	-
Pacific squadrons	95	-	99
Training squadrons	90	-	90

4. The average number of ready-hours per plane was only about 1/3 of the possible 720 hours per aircraft per month and again lowest for training squadrons:

	All	Atlantic	Pacific	Training
Average ready-hours per plane	240. 4	233. 4	247. 6	210. 4

5. All pairs of inputs had high degrees of substitutability.

6. The returns to scale parameter was always estimated as being below one, for both forms of the production function. This indicates that a proportional increase in all inputs will lead to a smaller proportional increase in the level of ready-hour production.

TABLE 4

CH-53 SQUADRONS

ALL

Number of observations: 55

Average value of data:

Ready-hours = 4182.2
Planes = 17.4
Maintenance man-hours = 3453.8
Spare parts = $136,797.4

Estimated production function

$$RH = 80.74\left[.62\,P^{-.08} + .09\,M^{-.02} + .26\,S^{-.07}\right]^{-10.50}$$

$R^2 = 0.685$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.978$
$\sigma_{PS} = 0.933$
$\sigma_{MS} = 0.972$

Estimated returns to scale parameter = 0.63

ATLANTIC

Number of observations: 20

Average value of data:

Ready-hours = 5251.6
Planes = 22.5
Maintenance man-hours = 3247.4
Spare parts = $75,598

Estimated production function

$$RH = 802.82\left[0.80\,P^{-0.08} + 0.22\,M^{-0.04} + 0.065^{-0.02}\right]^{-11.33}$$

$R^2 = 0.574$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.958$
$\sigma_{PS} = 0.979$
$\sigma_{MS} = 0.978$

Estimated returns to scale parameter = 0.34

PACIFIC

Number of observations: 34

Average value of data:

Ready-hours = 3540.7
Planes = 14.3
Maintenance man-hours = 3577.3
Spare parts = $173,808

Estimated production function

$$RH = 7.11\left[0.33\,P^{-0.03} + 0.19\,M^{-0.05} + 0.505^{-0.14}\right]^{-10.43}$$

$R^2 = 0.824$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.960$
$\sigma_{PS} = 0.927$
$\sigma_{MS} = 0.925$

Estimated returns to scale parameter = 0.73

TRAINING

Number of observations: 22

Average value of data:

Ready-hours = 5259.8
Planes = 25.0
Maintenance man-hours = 4900.8
Spare parts = $136,036

Estimated production function

$$RH = 177.02\left[0.80\,P^{-0.11} + 0.01\,M^{-0.001} + 0.285^{-0.06}\right]^{-10.20}$$

$R^2 = .411$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.999$
$\sigma_{PS} = 0.939$
$\sigma_{MS} = 0.999$

Estimated returns to scale parameter = 0.51

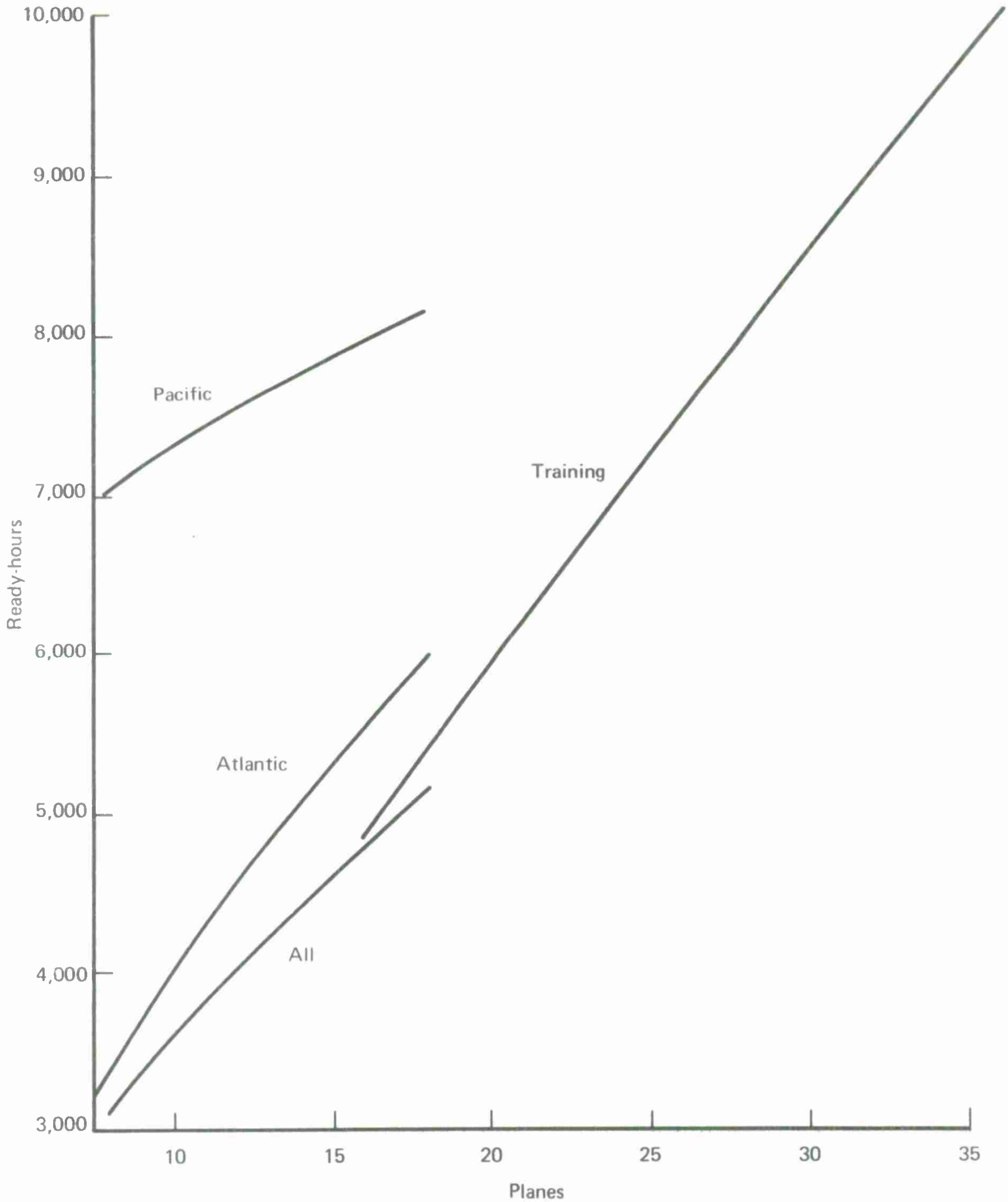


FIG. 2A: CH-53, PLANES

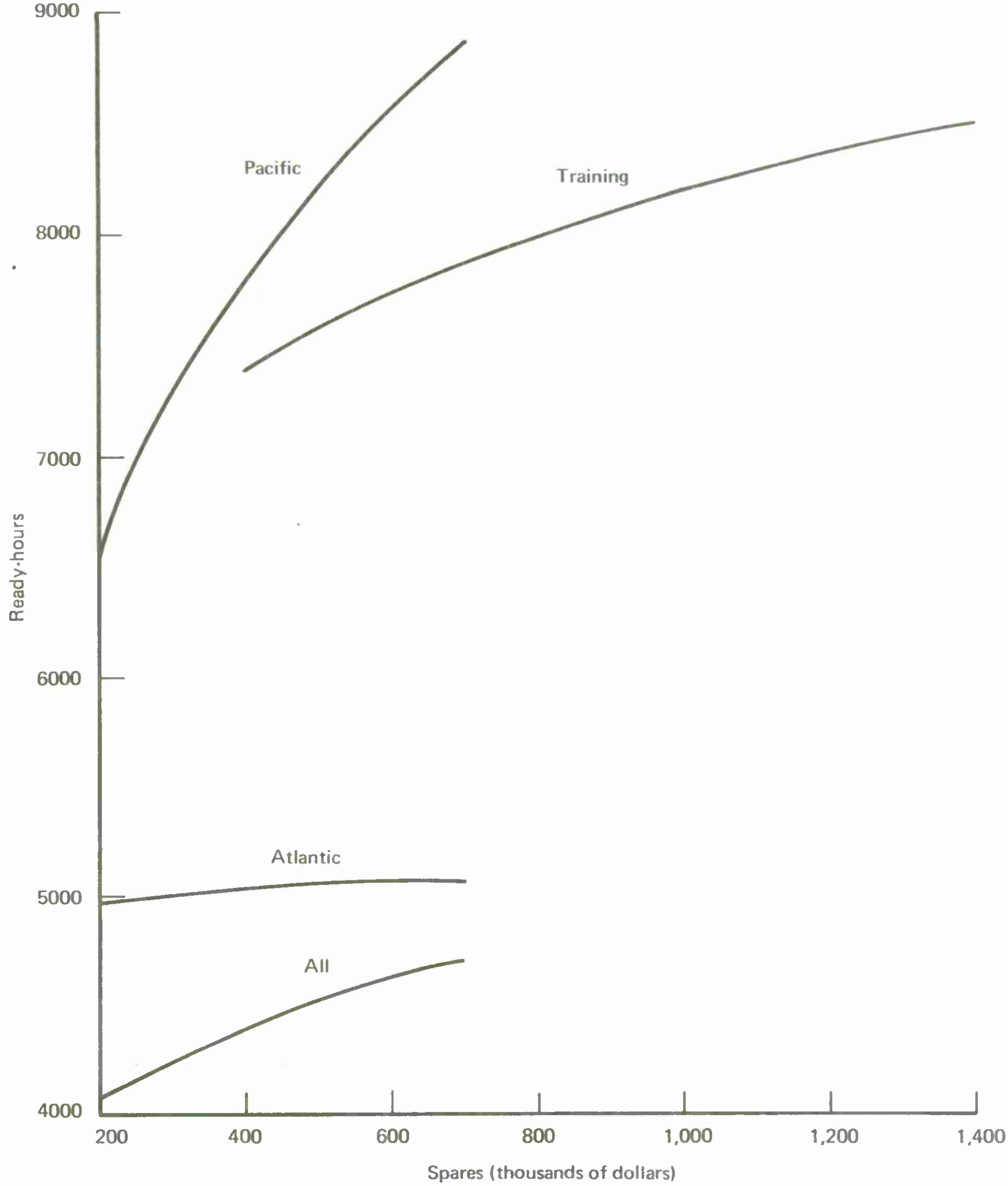


FIG. 2B: CH-53, SPARES

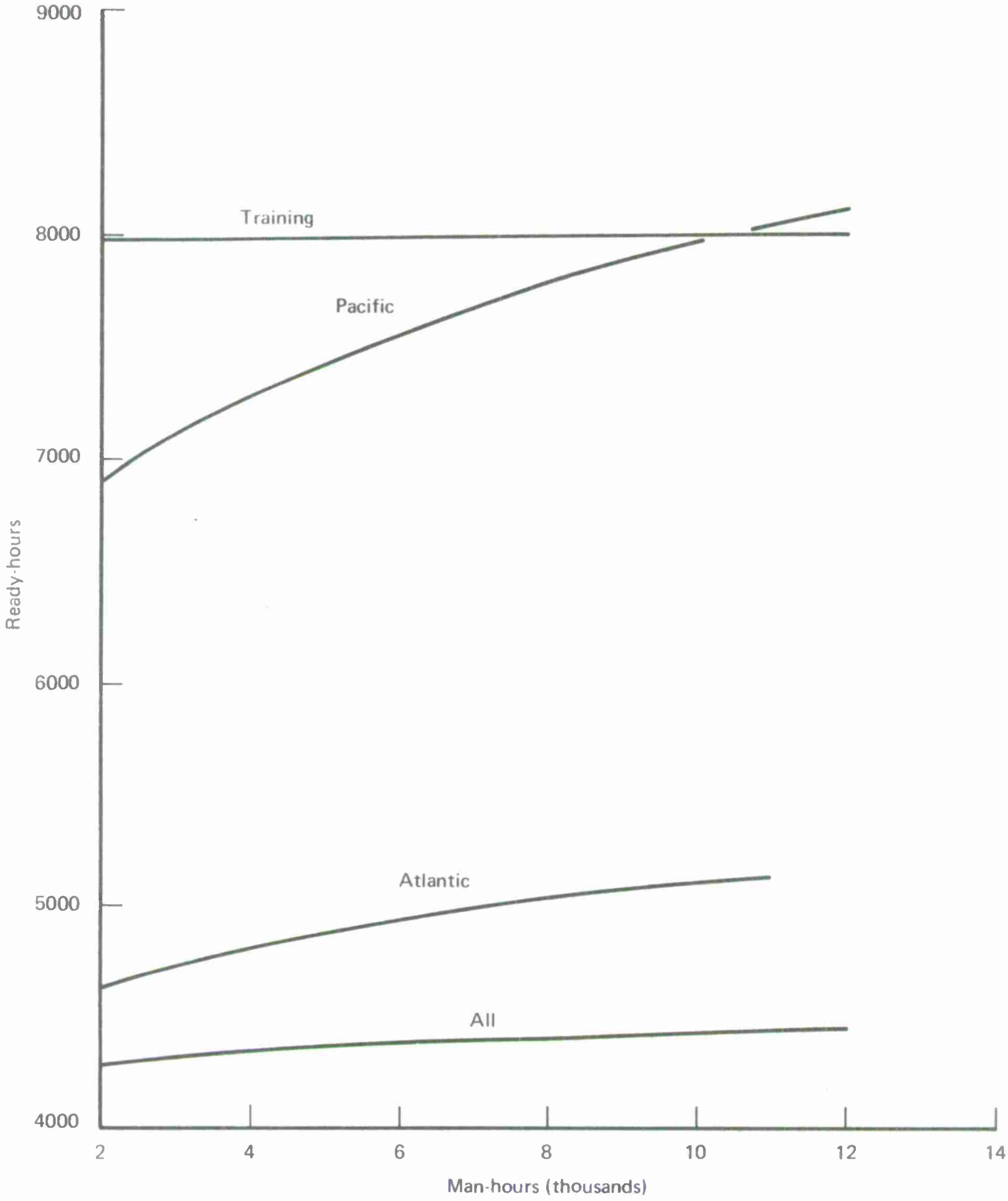


FIG. 2C: CH-53, MAINTENANCE MAN-HOURS

TABLE 5

CH-53 SQUADRONS

ALL

A		B		C	
RH = f(P, M, S)		RH = f(P, M, S)		RH = f(P, M, S)	
Planes	Ready-hours	Spares ($)	Ready-hours	Man hours	Ready hours
8	3110.4	200,000	4063.0	2,000	4308.4
9	3347.4	250,000	4170.7	2,500	4331.3
10	3574.2	300,000	4259.8	3,000	4349.9
11	3792.1	350,000	4335.9	3,500	4365.7
12	4002.2	400,000	4402.3	4,000	4379.4
13	4205.4	450,000	4461.3	4,500	4391.5
14	4402.3	500,000	4514.3	5,000	4402.3
15	4593.6	550,000	4562.6	5,500	4412.1
16	4779.9	600,000	4606.9	6,000	4421.0
17	4961.5	650,000	4647.8	6,500	4429.3
18	5138.7	700,000	4685.8	7,000	4436.9

ATLANTIC

8	3408.1	200,000	4982.6	2,000	4650.7
9	3699.7	250,000	4996.6	2,500	4740.4
10	3981.0	300,000	5007.9	3,000	4814.4
11	4253.3	350,000	5017.5	3,500	4877.6
12	4517.6	400,000	5025.8	4,000	4932.8
13	4774.9	450,000	5033.1	4,500	4981.7
14	5025.8	500,000	5039.6	5,000	5025.8
15	5270.8	550,000	5045.5	5,500	5065.8
16	5510.5	600,000	5050.9	6,000	5102.6
17	5745.3	650,000	5055.9	6,500	5136.5
18	5975.6	700,000	5060.5	7,000	5168.0

PACIFIC

8	7010.5	200,000	6571.6	2,000	6918.2
9	7167.3	250,000	6950.4	2,500	7122.6
10	7310.2	300,000	7268.3	3,000	7292.8
11	7441.9	350,000	7542.9	3,500	7438.9
12	7564.0	400,000	7784.9	4,000	7567.2
13	7678.0	450,000	8001.6	4,500	7681.6
14	7784.9	500,000	8197.9	5,000	7784.9
15	7885.8	550,000	8377.4	5,500	7879.3
16	7981.2	600,000	8543.0	6,000	7966.1
17	8071.8	650,000	8696.6	6,500	8046.5
18	8158.1	700,000	8840.0	7,000	8121.5

TRAINING

16	4806.4	400,000	7397.4	4,000	7993.7
18	5352.4	500,000	7586.9	5,000	7994.0
20	5891.8	600,000	7743.7	6,000	7994.2
22	6425.3	700,000	7877.7	7,000	7994.4
24	6953.4	800,000	7994.8	8,000	7994.5
26	7476.4	900,000	8098.9	9,000	7994.7
28	7994.8	1,000,000	8192.6	10,000	7994.8
30	8508.9	1,100,000	8277.9	11,000	7994.9
32	9018.9	1,200,000	8356.2	12,000	7995.0
34	9525.1	1,300,000	8428.6	13,000	7995.1
36	10027.7	1,400,000	8495.9	14,000	7995.2

RESULTS FOR THE S-2E

The results of the production function analysis for the S-2E antisubmarine aircraft are presented in the following tables and graphs, similar to those discussed in the preceding sections. Results are presented for the full cross-section of data and for Atlantic and Pacific squadrons, but an insufficient number of observations prohibited a separate estimation for training squadrons.

A summary of results for the S-2E, taken from the various tables and graphs, follows:

1. A large amount of the variance in actual ready-hour production is accounted for by the production function model. The R^2 statistic is in the range of 0.7.

2. The level of all 3 inputs to the production process appears to significantly affect the level of output produced. The number of planes is, of course, the most important of the 3 determinants.

3. The coefficients of the input parameters in the estimation of the Cobb-Douglas function are, in general, statistically different from zero in most cases, based upon the one-tailed t-test described earlier. The various levels of significance are shown below:

	Planes (percent)	Man-hours (percent)	Spare parts (percent)
All squadrons	99	90	75
Atlantic squadrons	99	75	-
Pacific squadrons	99	-	85

4. The average number of ready hours per aircraft is approximately 3/5 of the potential 720 hours per aircraft per month. This is seen below:

	All	Atlantic	Pacific
Average ready hours per plane	456.4	459.2	451.0

5. All pairs of inputs exhibit a high degree of substitutability.

6. The returns to scale parameter, estimated for both algebraic forms of the production function, is always less than one, again indicating a smaller proportionate output response to a proportionate change in the levels of all inputs.

TABLE 6

S-2E SQUADRONS

ALL

Number of observations: 54

Average value of data:

Ready-hours = 5430.964
Planes = 11.9
Maintenance man-hours = 1705.6
Spare parts = $138,601

Estimated production function

$$RH = 856.77\left[0.73\,P^{-0.10} + 0.27\,M^{-0.01} + 0.06\,S^{-0.12}\right]^{-\frac{0.74}{0.07}}$$

$R^2 = 0.661$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.978$
$\sigma_{PS} = 0.960$
$\sigma_{MS} = 0.972$

Estimated returns to scale parameter = 0.52

ATLANTIC

Number of observations: 25

Average value of data:

Ready-hours = 5602.4
Planes = 12.2
Maintenance man-hours = 2454.2
Spare parts = $141,072

Estimated production function

$$RH = 750.51\left[0.79\,P^{-0.08} + 0.24\,M^{-0.06} + 0.04\,S^{-0.03}\right]^{-9.44}$$

$R^2 = 0.741$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.941$
$\sigma_{PS} = 0.970$
$\sigma_{MS} = 0.968$

Estimated returns to scale parameter = 0.85

PACIFIC

Number of observations: 28

Average value of data:

Ready-hours = 5277.2
Planes = 11.7
Maintenance man-hours = 1034.5
Spare parts = $136,385

Estimated production function

$$RH = 574.27\left[0.81\,P^{-0.07} + 0.08\,M^{-0.02} + 0.11\,S^{-0.07}\right]^{-9.57}$$

$R^2 = 0.573$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.979$
$\sigma_{PS} = 0.935$
$\sigma_{MS} = 0.966$

Estimated returns to scale parameter = 0.67

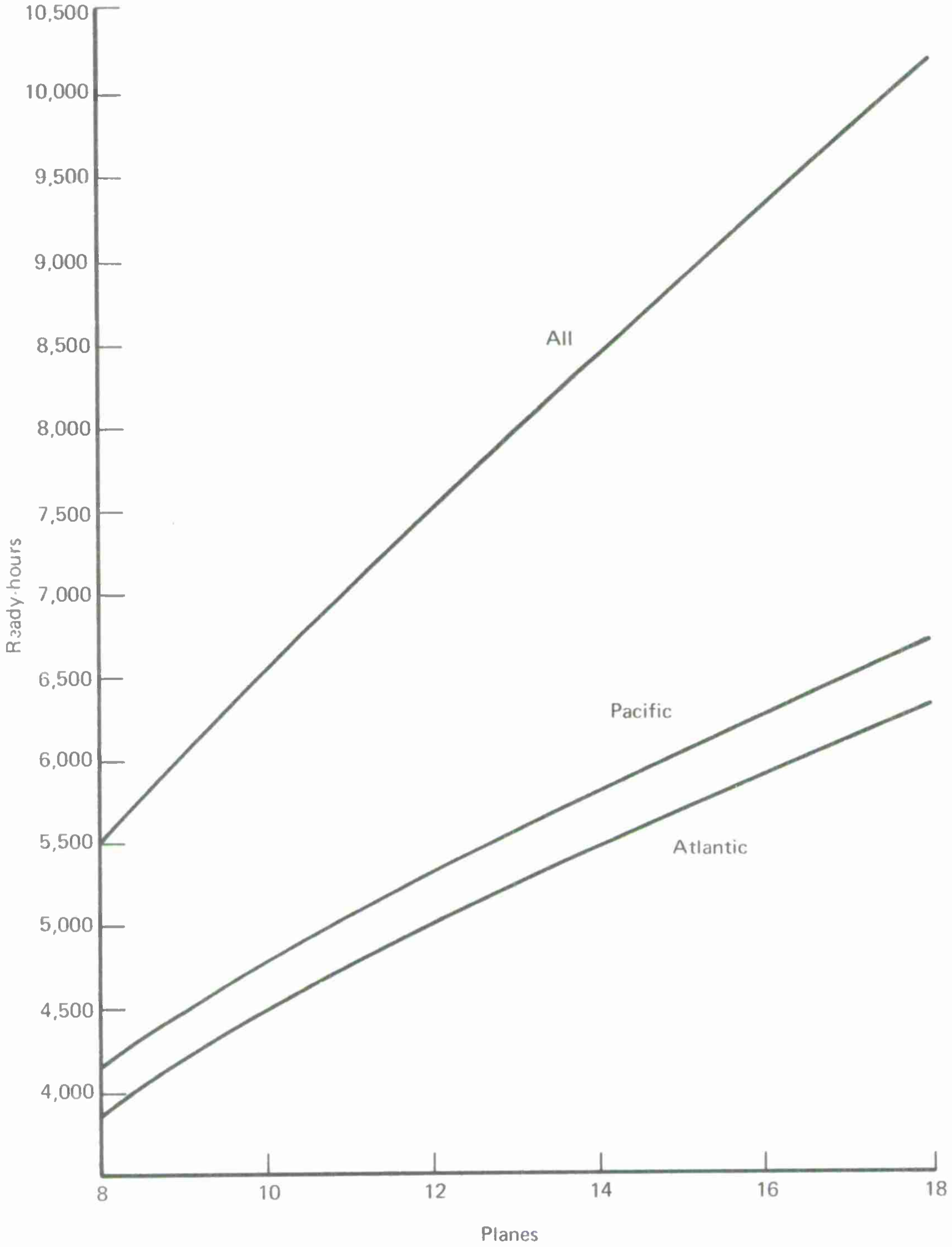


FIG. 3A: S-2E, PLANES

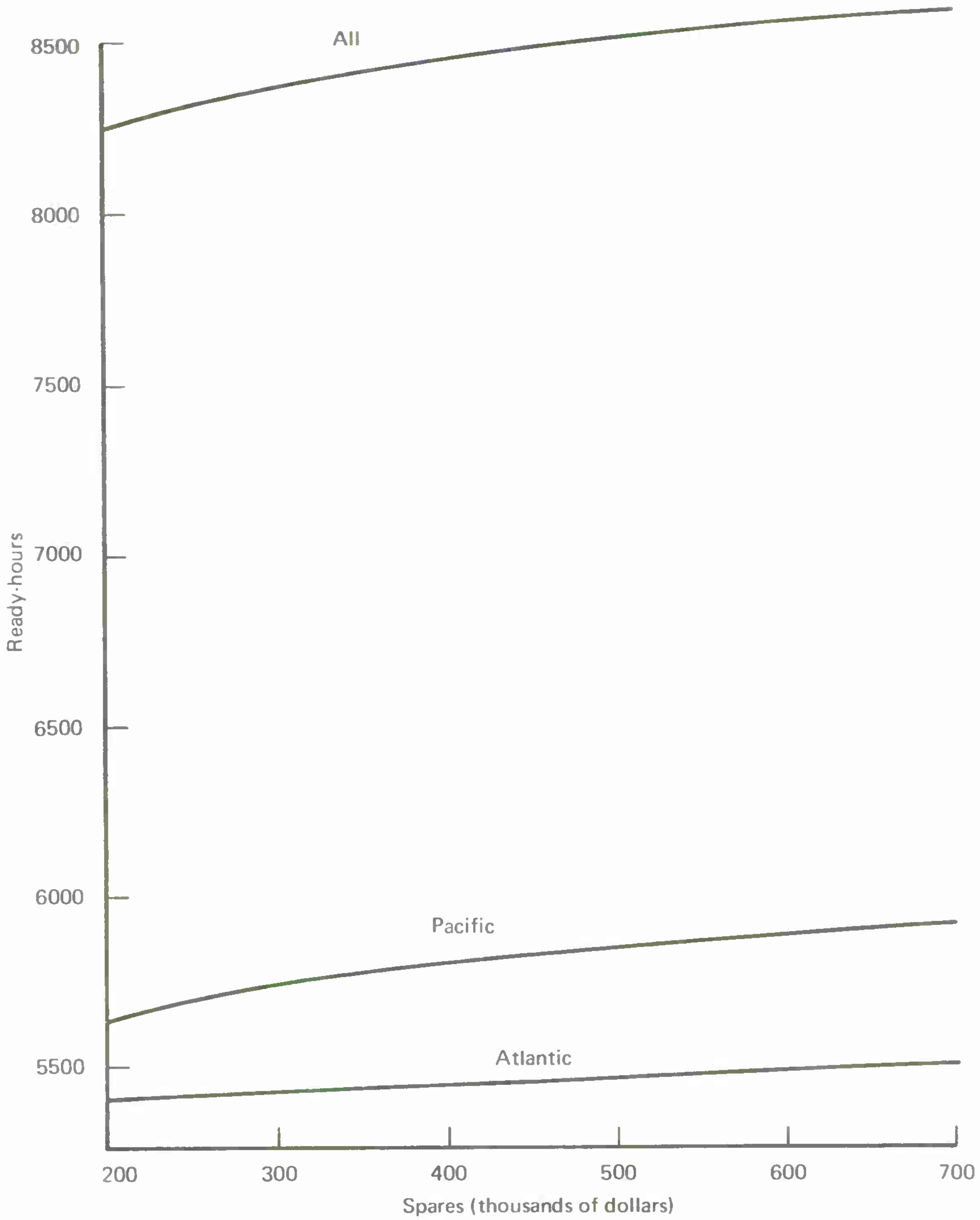


FIG. 3B: S-2E, SPARES

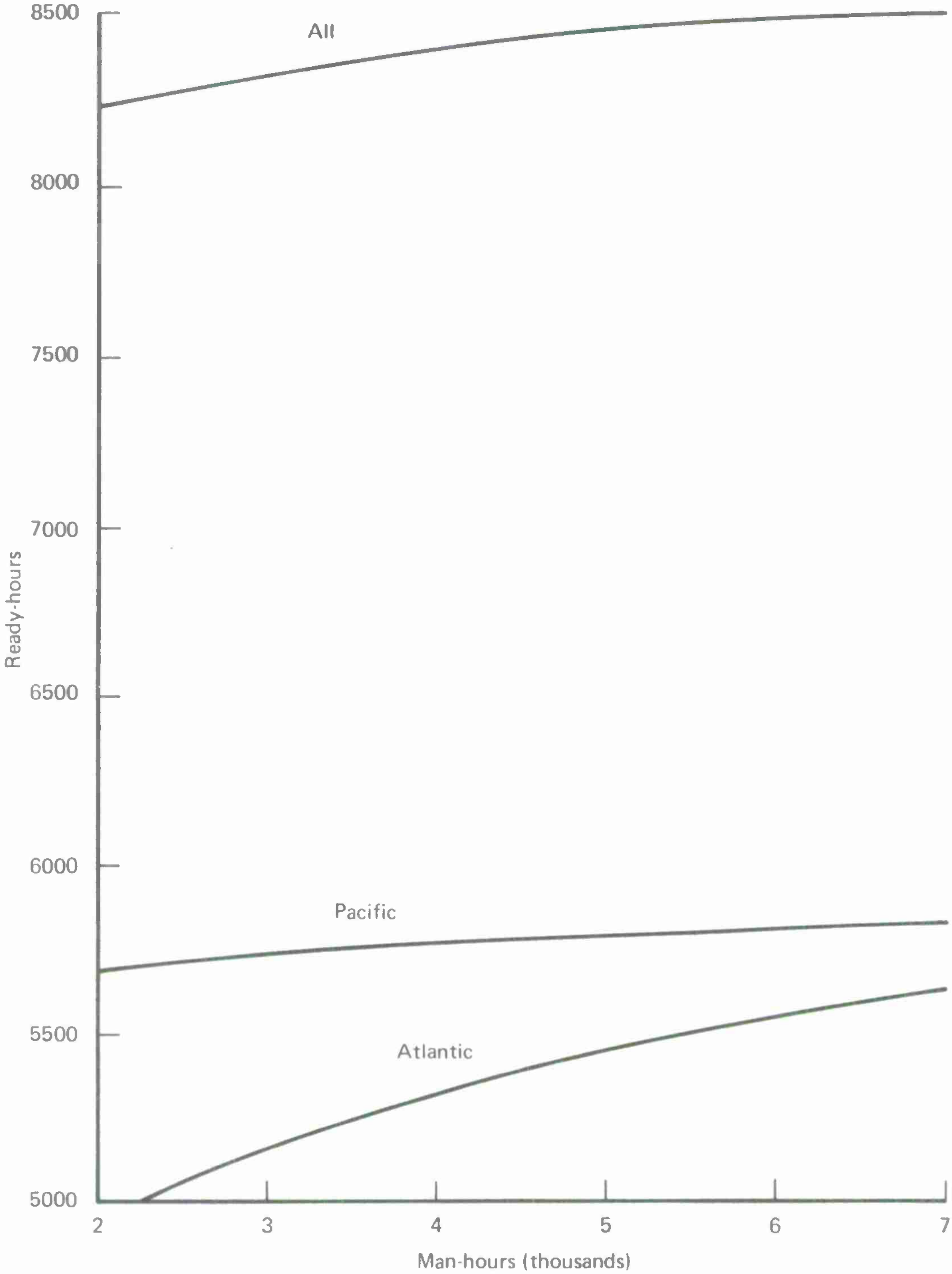


FIG. 3C: S-2E, MAINTENANCE MAN-HOURS

TABLE 7

S-2E SQUADRONS

ALL

A		B		C	
RH	f(P, M, S)	RH	f(P, M, S)	RH	f(P, M, S)
Planes	Ready hours	Spares (S)	Ready-hours	Man-hours	Ready-hours
8	5512.1	200,000	8256.5	2,000	8252.5
9	6031.0	250,000	8315.2	2,500	8297.7
10	6535.4	300,000	8363.2	3,000	8334.7
11	7027.2	350,000	8403.6	3,500	8366.1
12	7507.6	400,000	8438.7	4,000	8393.2
13	7977.7	450,000	8469.5	4,500	8417.2
14	8438.7	500,000	8497.1	5,000	8438.7
15	8891.1	550,000	8522.1	5,500	8458.1
16	9335.7	600,C00	8544.8	6,000	8475.8
17	9773.1	650,000	8565.8	6,500	8492.2
18	10203.8	700,000	8585.1	7,000	8507.3

ATLANTIC

8	3892.1	200,000	5405.7	2,000	4952.0
9	4177.7	250,000	5417.3	2,500	5069.0
10	4450.3	300,000	5426.8	3,000	5165.7
11	4711.7	350,000	5434.8	3,500	5248.2
12	4963.4	400,000	5441.7	4,000	5320.2
13	5206.4	450,000	5447.8	4,500	5384.2
14	5441.7	500,000	5453.2	5,000	5441.7
15	5670.0	550,000	5458.1	5,500	5494.0
16	5892.0	600,000	5462.6	6,000	5542.0
17	6108.2	650,000	5466.7	6,500	5586.3
18	6319.1	700,000	5470.5	7,000	5627.5

PACIFIC

8	4198.3	200,000	5640.9	2,000	5707.7
9	4493.7	250,000	5690.8	2,500	5729.0
10	4775.1	300,000	5731.5	3,000	5746.4
11	5044.6	350,000	5765.6	3,500	5761.1
12	5303.7	400,000	5795.1	4,000	5773.9
13	5553.5	450,000	5821.0	4,500	5785.1
14	5795.1	500,000	5844.2	5,000	5795.1
15	6029.3	550,000	5865.0	5,500	5804.2
16	6258.8	600,000	5884.0	6,000	5812.5
17	6478.1	650,000	5901.4	6,500	5820.1
18	6693.9	700,000	5917.5	7,000	5827.2

RESULTS FOR THE F-4B

The results of the production function analysis for the F-4B fighter aircraft are presented below in tables and graphs similar to those of the preceding sections. Along with the estimation based upon the full cross-section of data, estimates of the production function for Atlantic and Pacific squadrons are included; again, due to a lack of sufficient observations, no separate analysis of training squadrons was conducted.

A brief summary of the results for the F-4B appears below:

1. Approximately 3/5 of the total variation in actual ready-hour production is explained by the use of the model.

2. All 3 input variables contribute significantly to the prediction of ready-hour production. Of the 5 aircraft studied, the level of spare parts support appears to have the greatest effect, in terms of magnitude, on ready-hour production for the F-4B.

3. The co-efficients of the input variables in the Cobb-Douglas form of the production function are, in general, statistically different from zero, based upon the one-tailed t-test. The various levels of significance are shown below:

	Planes (percent)	Man-hours (percent)	Spare parts (percent)
All squadrons	90	95	99
Atlantic squadrons	75	-	99
Pacific squadrons	-	75	99

4. The average number of actual ready hours per aircraft is slightly less than half of the possible 720 hours per month:

	All	Atlantic	Pacific
Average ready hours per plane	338.3	336.8	339.2

5. All pairs of inputs exhibit a fairly high degree of substitutability, although the estimated elasticities are lower than those estimated for the other 4 aircraft.

6. The returns to scale parameter, estimated for both forms of the production function, is consistently estimated to be below one.

TABLE 8

F-4B SQUADRONS

ALL

Number of observations: 103

Average value of data:

Ready-hours = 4228.9
Planes = 12.5
Maintenance man-hours = 6216.5
Spare parts = $305,828

Estimated production function

$$RH = 122.63\left[0.67\,P^{-0.06} + 0.07\,M^{-0.22} + 0.46\,S^{-0.12}\right]^{-10.33}$$

$R^2 = 0.559$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.826$
$\sigma_{PS} = 0.905$
$\sigma_{MS} = 0.831$

Estimated returns to scale parameter = 0.62

ATLANTIC

Number of observations: 33

Average value of data:

Ready-hours = 4345.1
Planes = 12.9
Maintenance man-hours = 6389.8
Spare parts = $320,729

Estimated production function

$$RH = 115.49\left[0.59\,P^{-0.10} + 0.22\,M^{-0.06} + 0.32\,S^{-0.09}\right]^{-10.66}$$

$R^2 = 0.545$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.938$
$\sigma_{PS} = 0.916$
$\sigma_{MS} = 0.931$

Estimated returns to scale parameter = 0.64

PACIFIC

Number of observations: 69

Average value of data:

Ready-hours = 4172.5
Planes = 12.3
Maintenance man-hours = 6132.4
Spare parts = $298,591

Estimated production function

$$RH = 115.99\left[0.52\,P^{-0.03} + 0.18\,M^{-0.04} + 0.58\,S^{-0.16}\right]^{-10.75}$$

$R^2 = 0.592$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.964$
$\sigma_{PS} = 0.908$
$\sigma_{MS} = 0.929$

Estimated returns to scale parameter = 0.43

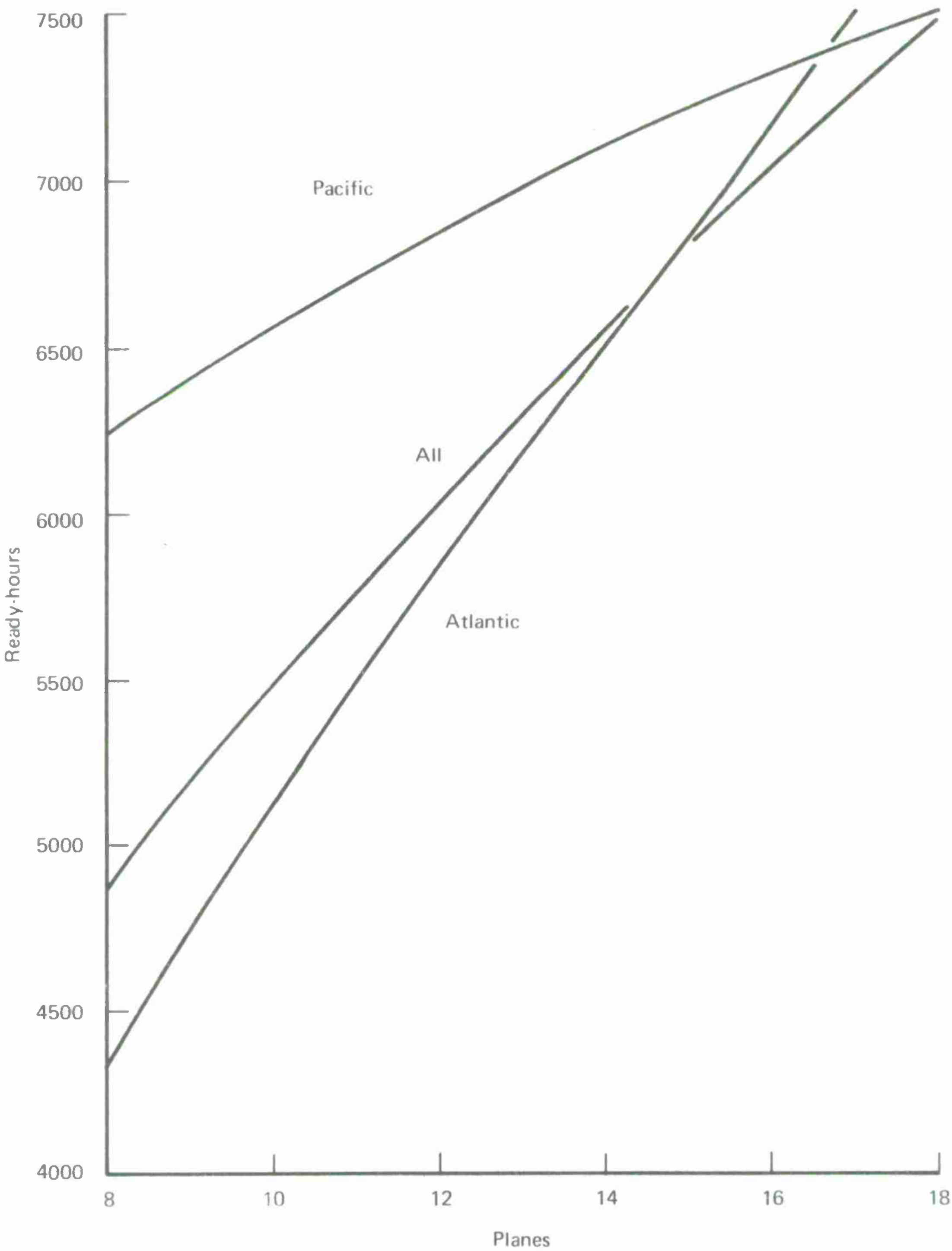


FIG. 4A: F-4B, PLANES

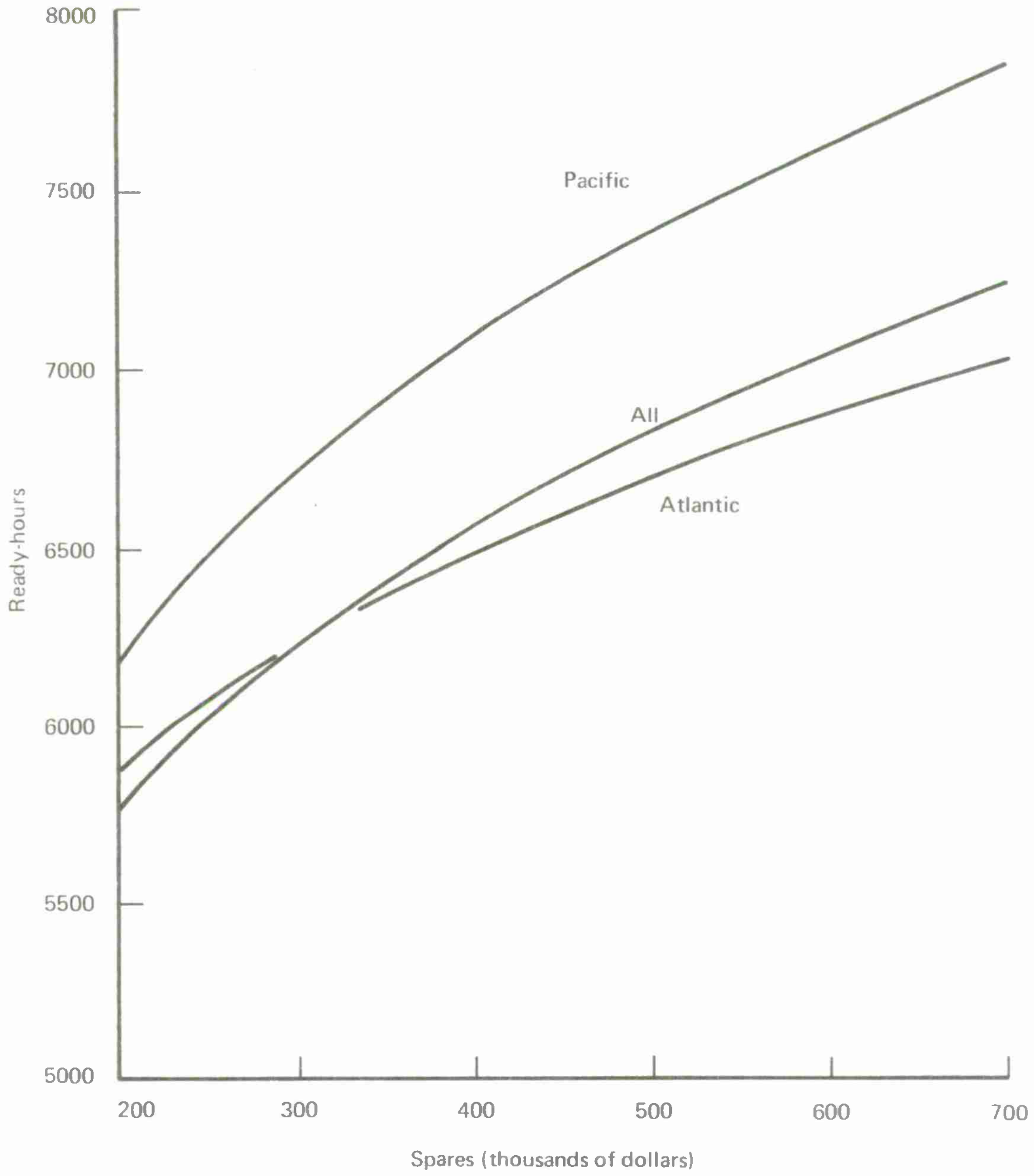


FIG. 4B: F-4B, SPARES

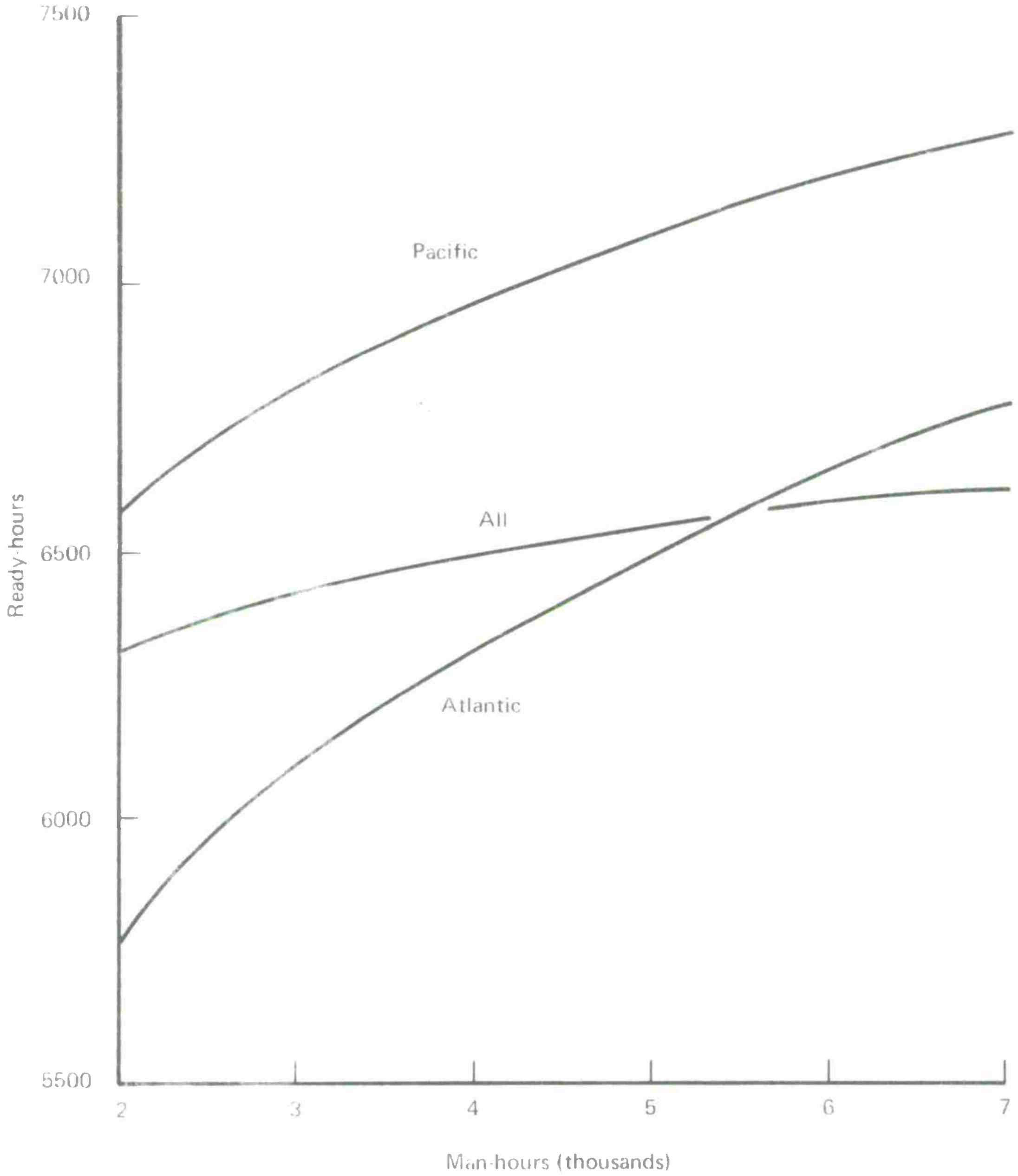


FIG. 4C: F-4B, MAINTENANCE MAN-HOURS

TABLE 9

F-4B SQUADRONS

ALL

A RH = f(P, M, S) Planes	 Ready-hours	B RH = f(P, M, S) Spares ($)	 Ready-hours	C RH = f(P, M, S) Man-hours	 Ready-hours
8	4889.5	200,000	5762.6	2,000	6315.9
9	5200.5	250,000	6011.1	2,500	6376.4
10	5495.1	300,000	6217.8	3,000	6424.1
11	5775.7	350,000	6394.8	3,500	6463.4
12	6044.1	400,000	6550.0	4,000	6496.4
13	6301.8	450,000	6688.1	4,500	6525.0
14	6550.0	500,000	6812.7	5,000	6550.0
15	6789.7	550,000	6926.3	5,500	6572.2
16	7021.6	600,000	7030.5	6,000	6592.2
17	7246.6	650,000	7127.0	6,500	6610.2
18	7465.2	700,000	7216.8	7,000	6626.8

ATLANTIC

8	4363.5	200,000	5881.1	2,000	5790.6
9	4748.1	250,000	6076.9	2,500	5958.5
10	5119.4	300,000	6239.2	3,000	6097.9
11	5478.9	350,000	6377.9	3,500	6217.4
12	5828.1	400,000	6499.3	4,000	6322.0
13	6167.9	450,000	6607.1	4,500	6415.2
14	6499.3	500,000	6704.3	5,000	6499.3
15	6822.9	550,000	6792.7	5,500	6575.9
16	7139.6	600,000	6873.9	6,000	6646.3
17	7449.7	650,000	6948.9	6,500	6711.5
18	7753.8	700,000	7018.7	7,000	6772.2

PACIFIC

8	6241.7	200,000	6193.0	2,000	6577.3
9	6411.7	250,000	6478.1	2,500	6700.0
10	6567.8	300,000	6713.8	3,000	6801.3
11	6711.7	350,000	6914.8	3,500	6887.7
12	6845.9	400,000	7090.0	4,000	6963.0
13	6971.6	450,000	7245.5	4,500	7029.9
14	7090.0	500,000	7385.2	5,000	7090.0
15	7202.0	550,000	7512.0	5,500	7144.7
16	7308.2	600,000	7628.1	6,000	7194.8
17	7409.4	650,000	7735.2	6,500	7241.1
18	7506.1	700,000	7834.6	7,000	7284.1

RESULTS FOR THE TA-4F

The results of the production function analysis for the TA-4F attack trainer are summarized in the tables and graphs below. Separate results for Atlantic and Pacific squadrons are presented along with the estimates based upon the full cross-section of data.

A summary of the results and conclusions taken from these tables is given below:

1. A large percentage of the variation in observed ready-hour production is explained by the production function model. The R^2 statistic is in the range of 0.8, the highest level obtained for any of the 5 aircraft studied.

2. The level of maintenance employed by the squadrons appears to have little effect on ready-hour production; the parameters associated with this variable are all estimated to be very close to zero. The levels of spare parts and planes, however, both appear to be quite important.

3. In the estimation of the Cobb-Douglas form of the production function, the coefficients of planes and spare parts are all statistically significant, as measured by the standard one-tailed t-test described earlier. The coefficients of man-hours in some cases were, actually estimated to be negative, although not significantly so. These results are summarized in the table of significance levels below:

	Planes (percent)	Man-hours	Spare parts (percent)
All squadrons	99	-	95
Atlantic squadrons	99	-	85
Pacific squadrons	99	-	75

4. The average number of ready hours per plane is slightly less than half of the possible 720 hours per plane per month:

	All	Atlantic	Pacific
Average ready hours per plane	336.1	343.1	334.3

5. The degree of substitutability of the various input pairs is estimated to be quite high, as in the previous cases.

6. The return to scale parameter is again estimated to be below one, taking on values in the very low range of 0.3. This suggests that, for example, a 10 percent increase in the levels of all inputs would lead to only a 3 percent increase in ready-hour production.

TABLE 10

TA-4F SQUADRONS

ALL

Number of observations: 73

Average value of data:

Ready-hours = 3462.3
Planes = 10.3
Maintenance man-hours = 1583.5
Spare parts = $63,793

Estimated production function

$$RH = 232.58\left[.650^{-.06} + .07\,M^{-.02} + .52\,S^{-.12}\right] - 10.00$$

$R^2 = .814$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.979$
$\sigma_{PS} = 0.907$
$\sigma_{MS} = 0.973$

Estimated returns to scale parameter = 0.40

ATLANTIC

Number of observations: 28

Average value of data:

Ready-hours = 3602.7
Planes = 10.5
Maintenance man-hours = 1851.9
Spare parts = $71,628

Estimated production function

$$RH = 112.83\left[.67\,P^{-.08} + 0.02\,M^{-0.01} + .23\,S^{-0.05}\right] - 10.66$$

$R^2 = .843$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.990$
$\sigma_{PS} = 0.949$
$\sigma_{MS} = 0.989$

Estimated returns to scale parameter = 0.64

PACIFIC

Number of observations: 45

Average value of data:

Ready-hours = 3376.8
Planes = 10.1
Maintenance man-hours = 1420.2
Spare parts = $59,024

Estimated production function

$$RH = 608.15\left[0.71\,P^{-0.06} + 0.13\,M^{-0.03} + 0.57\,S^{-0.15}\right] - 10.50$$

$R^2 = 0.793$

Pairwise elasticities of substitution

$\sigma_{PM} = 0.969$
$\sigma_{PS} = 0.887$
$\sigma_{MS} = 0.949$

Estimated returns to scale parameter = 0.21

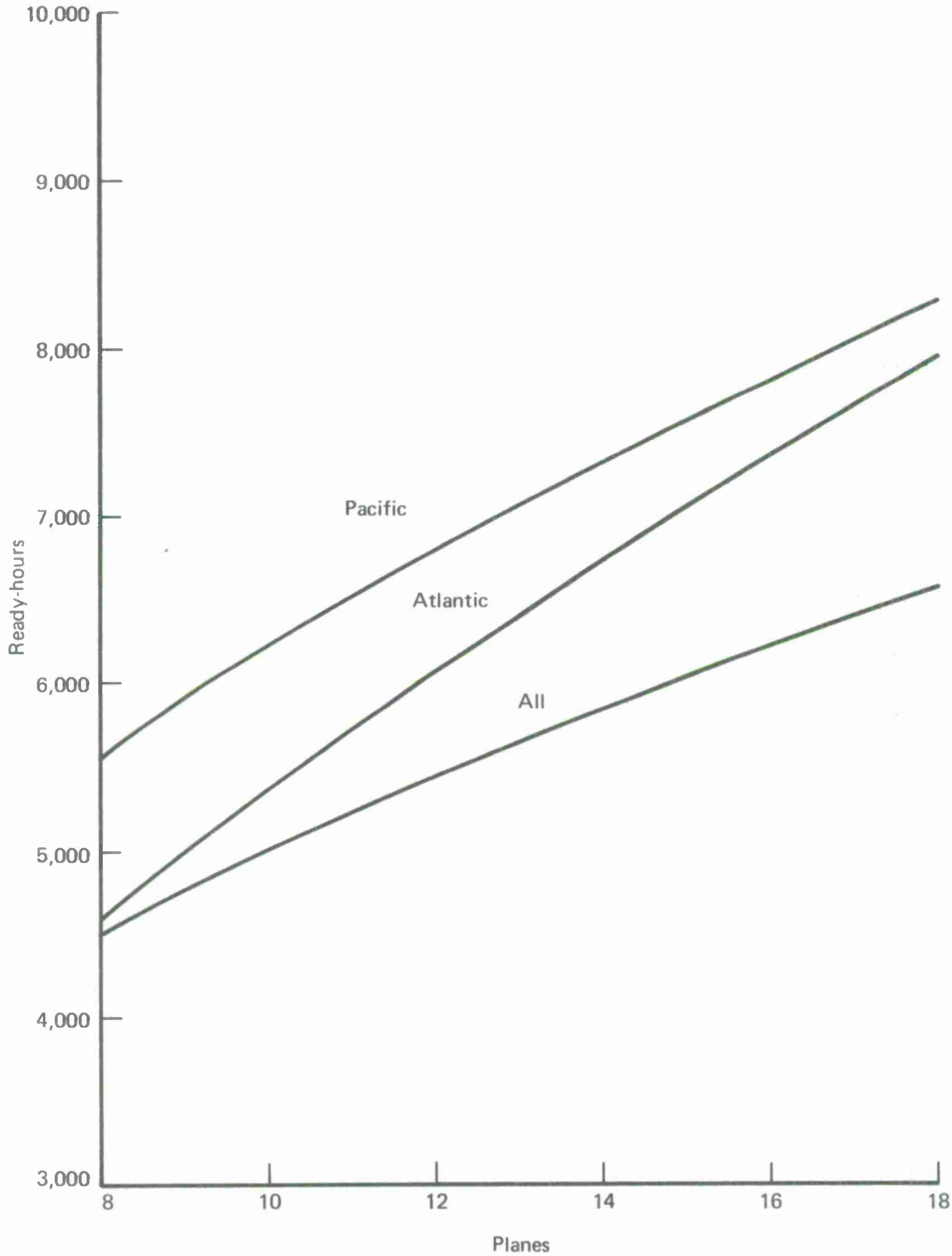


FIG. 5A: TA-4F, PLANES

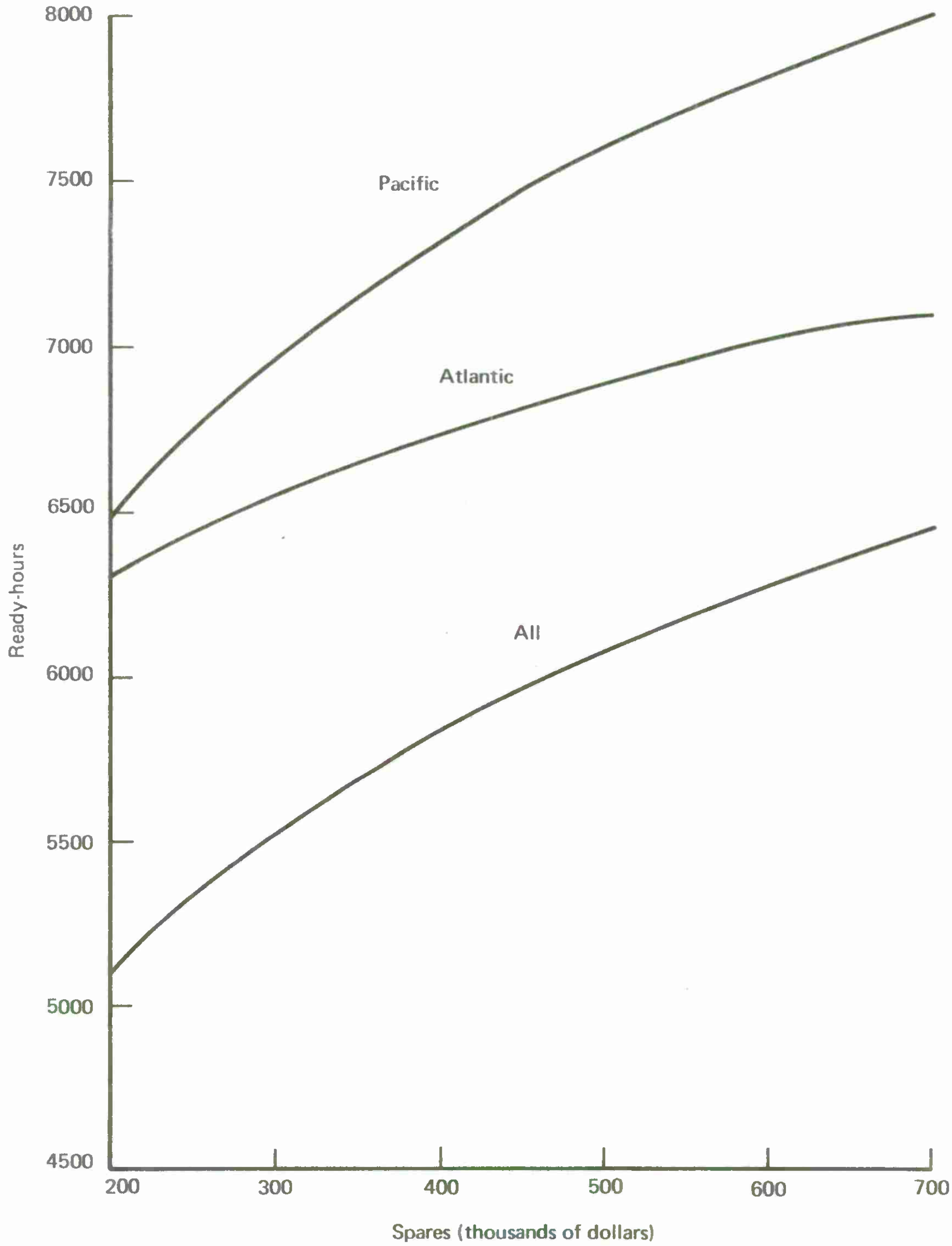


FIG. 5B: TA-4F, SPARES

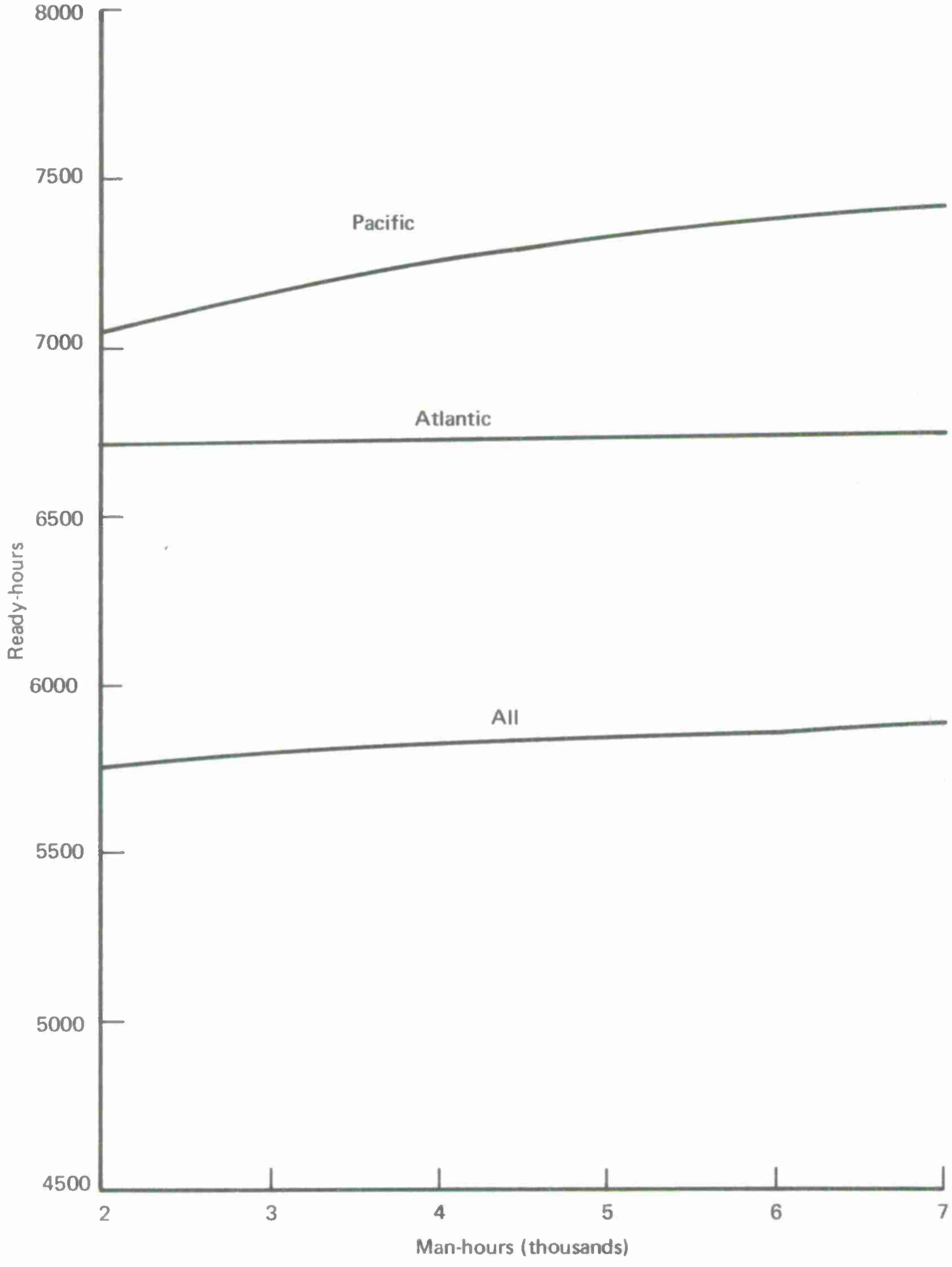


FIG. 5C: TA-4F, MAINTENANCE MAN-HOURS

TABLE 11

TA-4F SQUADRONS

ALL

A RH = f(P, M, S) Planes	 Ready-hours	B RH = f(P, M, S) Spares ($)	 Ready-hours	C RH = f(P, M, S) Man-hours	 Ready-hours
8	4512.2	200,000	5121.2	2,000	5753.8
9	4764.9	250,000	5348.1	2,500	5775.1
10	5002.6	300,000	5537.0	3,000	5792.5
11	5227.5	350,000	5699.0	3,500	5807.2
12	5441.3	400,000	5841.2	4,000	5819.9
13	5645.5	450,000	5967.8	4,500	5831.1
14	5841.2	500,000	6082.1	5,000	5841.2
15	6029.2	550,000	6186.3	5,500	5850.2
16	6210.4	600,000	6282.0	6,000	5858.5
17	6385.4	650,000	6370.7	6,500	5866.1
18	6554.8	700,000	6453.2	7,000	5873.2

ATLANTIC

8	4600.3	200,000	6306.4	2,000	6721.5
9	4986.7	250,000	6444.0	2,500	6725.9
10	5359.1	300,000	6557.7	3,000	6729.4
11	5719.2	350,000	6654.7	3,500	6732.4
12	6068.6	400,000	6739.3	4,000	6735.0
13	6408.3	450,000	6814.5	4,500	6737.3
14	6739.3	500,000	6882.2	5,000	6739.3
15	7062.5	550,000	6943.7	5,500	6741.2
16	7378.4	600,000	7000.2	6,000	6742.9
17	7687.8	650,000	7052.3	6,500	6744.4
18	7991.0	700,000	7100.8	7,000	6745.9

PACIFIC

8	5576.9	200,000	6495.5	2,000	7051.2
9	5906.3	250,000	6758.1	2,500	7116.0
10	6216.9	300,000	6974.5	3,000	7169.2
11	6511.6	350,000	7158.7	3,500	7214.3
12	6792.4	400,000	7319.0	4,000	7253.4
13	7061.0	450,000	7460.9	4,500	7288.0
14	7319.0	500,000	7588.2	5,000	7319.0
15	7567.3	550,000	7703.7	5,500	7347.0
16	7807.0	600,000	7809.3	6,000	7372.7
17	8038.9	650,000	7906.6	6,500	7396.3
18	8263.6	700,000	7996.9	7,000	7418.2

RELATIONSHIP BETWEEN SPARES USAGE AND NORS RATES

In the following section we will show how the NORS rate can be simply calculated for a type/model/series from the production function.

The relationships between spares usage and ready hours can be determined with the production function by varying the spares usage while holding planes and man-hours constant at some specified level. The constant values that are usually used are either the averages over the sample range or current planning factors. It should be noted that the derived relationship between spares usage and ready hours is not the optimal relationship determined in the cost function analysis. Given the above conditions, the increase in ready hours from increasing spares usage will come from the reduction in NORS hours. This, of course, will result in a reduction of the NORS rate.

Squadron ready hours in a month is equal to total available hours (number of planes x 720 hours) minus hours in scheduled maintenance minus NORS hours plus NORM hours. Since we are holding planes constant, total available hours will be constant. Also, we would expect NORM hours to remain stable when spares usage is increased. This is due to the fact that NORM hours is mainly dependent on man-hours, which is not being varied. Hence, the increase in ready hours due to an increase in spares usage will come almost entirely from a reduction in NORS hours. The results for the F-4B, CH-53, and TA-4F are shown in tables 12, 13, and 14. The graphs of the relationship between the NORS rate and spares usage for the three type/model/series are shown in the figures beneath the tables.

The NORS hours, NORS rate, and spares usage in the first row of each table are the averages obtained from the sample data. The ready hours in the first row were derived by using the average values of the 3 resources. The following rows show the increase in ready hours and the reduction in the NORS rates resulting from an increase in spares usage.

For example, an F-4B squadron composed of 12 aircraft and utilizing 6216 man-hours in a month, will increase its ready hours from 5928.1 to 6221.3 if spares usage is increased from $305,828 to $400,000. This will reduce NORS hours from 984.2 to 690.9 and the NORS rate from 11.4 percent to 10.8 percent.

TABLE 12

AIRCRAFT: F-4B
Average value of the data
Planes = 12; man-hours = 6216.5

Ready-hours	NORS hours	NORS rate (percent)	Spares usage (dollars)
5928.1	984.2	11.4	305,828
5977.1	935.2	10.8	320,000
6042.9	869.3	10.1	340,000
6105.4	806.8	9.3	360,000
6221.3	690.9	7.9	400,000
6326.8	585.4	6.7	440,000
6469.6	442.7	5.1	500,000

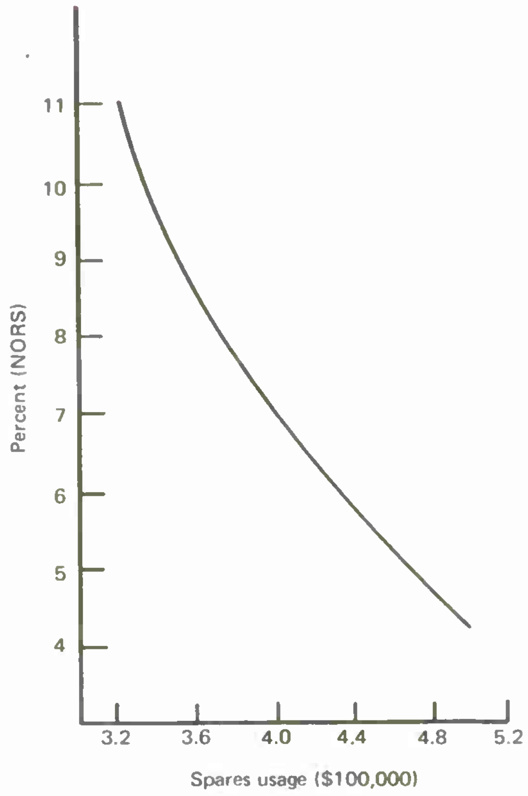


TABLE 13

AIRCRAFT: CH-53
Average value of the data:
Planes = 17; man-hours = 3453.8

Ready-hours	NORS hours	NORS rate (percent)	Spares usage (dollars)
4332.8	3084.4	25.2	136,979
4535.2	2882.0	23.5	200,000
4656.8	2760.4	22.6	250,000
4757.4	2659.8	21.7	300,000
4918.2	2499.0	20.4	400,000
5044.7	2372.5	19.4	500,000
5149.2	2268.0	18.5	600,000
5238.4	2178.8	17.8	700,000
5385.2	2032.0	16.6	900,000
5447.3	1969.9	16.1	1,000,000

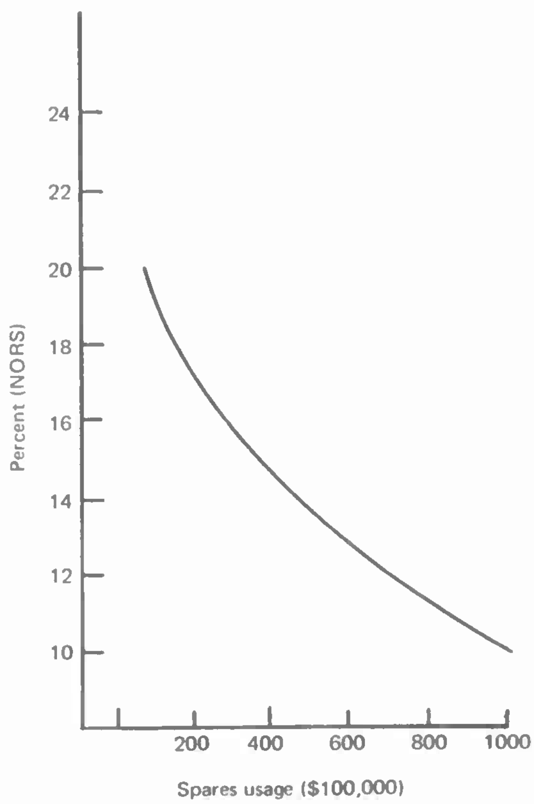
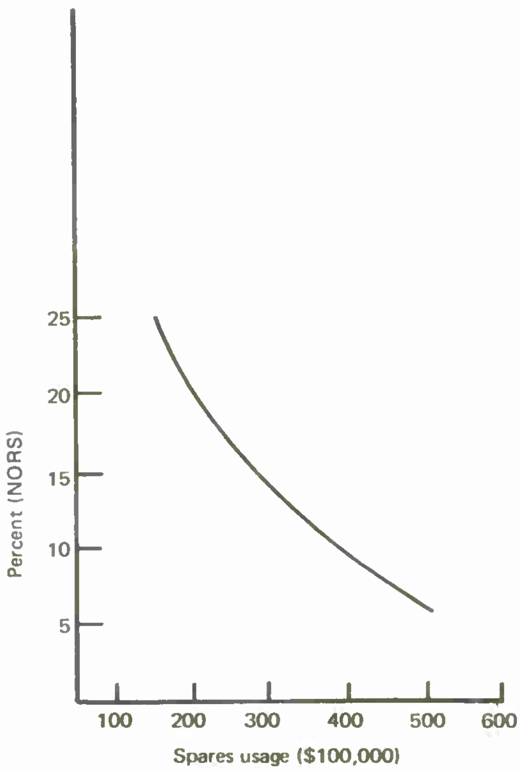


TABLE 14

AIRCRAFT: TA-4F
Average value of the data:
Planes = 10; man-hours = 1583. 5

Ready-hours	NORS hours	NORS rate (percent)	Spares usage (dollars)
3415. 1	1873. 3	26. 0	63, 792
3756. 0	1532. 4	21. 3	100, 000
4314. 6	973. 8	13. 5	200, 000
4658. 7	629. 7	8. 7	300, 000
4910. 2	378. 2	5. 3	400, 000



RESULTS OF THE COST FUNCTION ANALYSIS

In this section, the results of the cost function analysis for the F-4B fighter, CH-53 helicopter, and TA-4F attack trainer are presented and discussed. The general methodology discussed earlier was extended to determine the optimal mixture of resources to be supplied to a squadron given various funding levels for resource usage. The following non-linear optimization model was used in this analysis:

$$\max_{P,M,S} \quad RH = A\left[\alpha_1 P^{-\rho_1} + \alpha_2 M^{-\rho_2} + \alpha_3 S^{-\rho_3}\right]^{-\frac{1}{\rho}}$$

$$\text{subject to } p_1 P + p_2 M + p_3 S = C \ ,$$

where

RH = ready hours

P = number of planes

M = number of man-hours

S = dollar value of spares usage

C = budget for resource usage

P_i = per unit prices of the various resources (see appendix A for price derivation methodology),

and the other parameters are those resulting from the production function analysis. The derivation of optimal resource combinations required 2 separate stages of analysis — the empirical estimation of the production functions, and the solution of the maximization problem given above. Thus the results that follow may be considered to reflect all current operating doctrines, in that the production function estimation was based upon actual data collected from the 3M system. Therefore, the resulting optimization is based upon the ways in which the usage of the various resources is currently seen to affect ready-hour production.

The use of LaGrangian multiplier techniques similar to those developed in the preceding section yields the following equations, which can be solved for the optimal levels of planes (P*), man-hours (M*), and spare parts usage (S*):

$$p_1 P^* + p_2 \left[\left(\frac{p_2}{p_1} \frac{\alpha_1 \rho_1}{\alpha_2 \rho_2} \right) \left(P^* \right)^{-(1+\rho_1)} \right]^{-\frac{1}{1+\rho_2}} \tag{1}$$

$$+ p_3 \left[\left(\frac{p_3}{p_1} \frac{\alpha_1 \rho_1}{\alpha_3 \rho_3} \right) \left(P^* \right)^{-(1+\rho_1)} \right]^{-\frac{1}{1+\rho_3}} = C$$

$$M^* = \left[\left(\frac{p_2}{p_1} \frac{\alpha_1 \rho_1}{\alpha_2 \rho_2} \right) \left(P^* \right)^{-(1+\rho_1)} \right]^{-\frac{1}{1+\rho_2}} \tag{2}$$

$$S^* = \left[\left(\frac{p_3}{p_1} \frac{\alpha_1 \rho_1}{\alpha_3 \rho_3} \right) \left(P^* \right)^{-(1+\rho_1)} \right]^{-\frac{1}{1+\rho_3}} \tag{3}$$

Equation (1) , which may be solved for P^* , is highly non-linear. However, since the derivative of the left hand side of (1) with respect to P^* is seen to be

$$p_1 + p_2 \left[-\frac{1}{1+\rho_2} \right] \left[\left(\frac{p_2}{p_1} \frac{\alpha_1 \rho_1}{\alpha_2 \rho_2} \right) \left(P^* \right)^{-(1+\rho_1)} \right]^{-\frac{1}{1+\rho_2} - 1} \left[\frac{p_2}{p_1} \frac{\alpha_1 \rho_1}{\alpha_2 \rho_2} \right] \left[-(1+\rho_1) \right] \left[\left(P^* \right)^{-(1+\rho_1) - 1} \right]$$

$$+ p_3 \left[-\frac{1}{1+\rho_3} \right] \left[\left(\frac{p_3}{p_1} \frac{\alpha_1 \rho_1}{\alpha_3 \rho_3} \right) \left(P^* \right)^{-(1+\rho_1)} \right]^{-\frac{1}{1+\rho_3} - 1} \left[\frac{p_3}{p_1} \frac{\alpha_1 \rho_1}{\alpha_3 \rho_3} \right] \left[-(1+\rho_1) \right] \left[\left(P^* \right)^{-(1+\rho_1) - 1} \right] > 0 ,$$

the value of P^* solving the equation is found to be unique and easily determined through iterative procedures. These procedures were programmed and used to produce the results summarized in this section. Given the value of P^* , the values of M^* and S^* can be determined through the use of equations (2) and (3).

Finally, the expected level of ready hours resulting from the optimal input combinations can be determined by use of the equation

$$RH^* = A \left[\alpha_1 (P^*)^{-\rho_1} + \alpha_2 (M^*)^{-\rho_2} + \alpha_3 (S^*)^{-\rho_3} \right]^{-\frac{1}{\rho}} , \qquad (4)$$

where the parameter values are those determined by the empirical estimation described earlier.

The results of the analysis are displayed below in a number of tables and graphs. One major result of the analysis, however, requires separate mention.

- First, for all of the aircraft analyzed, current levels of spare parts usage are found to be below the optimal level for the same total resource budget. Two primary conclusions are drawn from the fact. First, if current total resource budgets are retained, a larger percentage of those budgets should be allocated to the spares categories. For the 3 aircraft studied, the recommended increases in the spare parts usage budgets are 25 percent for the F-4B, 66 percent for the CH-53, and 11 percent for the TA-4F. Substantially higher levels of readiness could be expected in each of these cases. These results are summarized in table 15.

- Second, even if squadron size, in terms of the number of planes, is to be maintained in the future, the results indicate the need for more spare parts support. These results are summarized in the remaining tables in this section.

These are significant results. Considering either present or optimal squadron configurations, the need for higher levels of spare parts usage is noted. In this sense, a "balancing" of the various resource category budgets requires a greater proportional allocation to the spare parts categories.

TABLE 15

COMPARISON OF PRESENT AND OPTIMAL ALLOCATIONS OF RESOURCES UNDER THE SAME TOTAL BUDGET

F-4B

	Planes	Man-hours	Spares (dollars)	Ready-hours
Present*	12	6216.5	305,828	4229
Optimum**	11	5837.7	382,884	5963

CH-53

	Planes	Man-hours	Spares (dollars)	Ready-hours
Present*	17	3453.8	136,979	4182
Optimum**	15	8723.5	227,451	4504

TA-4F

	Planes	Man-hours	Spares (dollars)	Ready-hours
Present*	10	1583.5	63,792	3462
Optimum**	10	956.7	70,932	3473

*Determined from empirical analysis of actual data.
**Determined with the use of the cost function model.

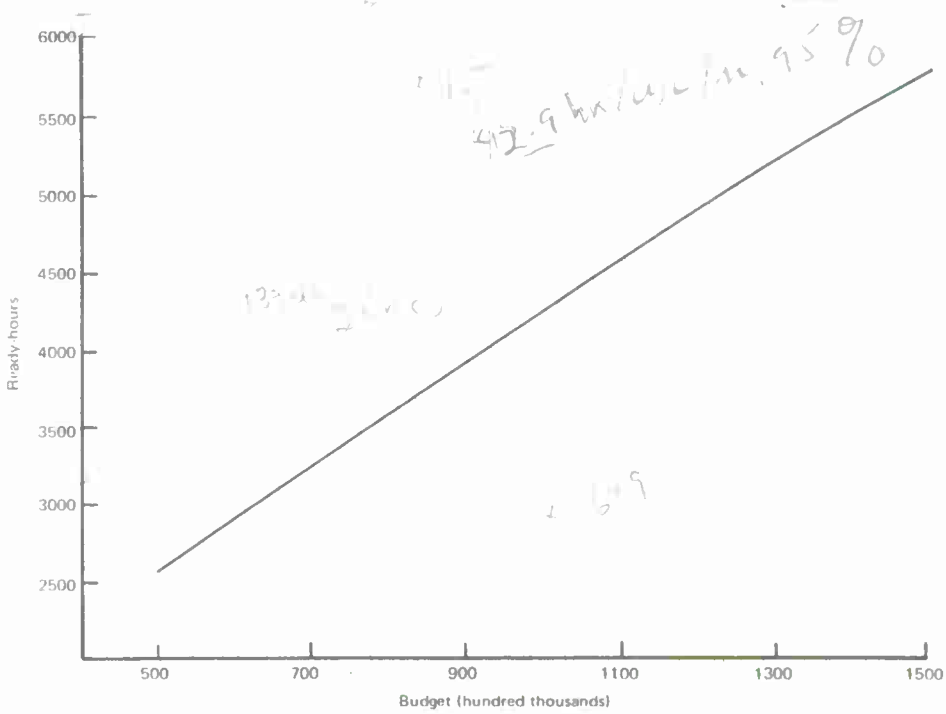
TABLE 16

OPTIMAL F-4B SQUADRONS FOR VARIOUS TOTAL USAGE BUDGET LEVELS

Optimization Results

Cost (dollars)	Planes	Man-hours	Spares (dollars)	Ready hours
500,000	3.63	2103.00	125,915.03	2548.57
550,000	4.00	2288.52	138,060.63	2735.32
600,000	4.37	2472.11	150,167.48	2917.55
650,000	4.74	2653.96	162,238.98	3095.73
700,000	5.11	2834.22	174,277.55	3270.25
750,000	5.49	3013.00	186,285.58	3441.43
800,000	5.86	3190.42	198,265.30	3609.54
850,000	6.23	3366.57	210,218.20	3774.82
900,000	6.61	3541.54	222,146.41	3937.48
950,000	6.98	3715.39	234,050.57	4097.70
1,000,000	7.36	3888.19	245,932.54	4255.64
1,050,000	7.73	4060.00	257,793.41	4411.44
1,100,000	8.11	4230.88	269,633.85	4565.22
1,150,000	8.49	4400.86	281,454.97	4717.10
1,200,000	8.86	4569.99	293,257.53	4867.18
1,250,000	9.24	4738.32	305,042.54	5015.56
1,300,000	9.62	4905.87	316,810.55	5162.32
1,350,000	9.99	5072.68	328,562.28	5307.54
1,400,000	10.37	5238.78	340,298.01	5451.29
1,450,000	10.75	5404.20	352,018.94	5593.63
1,500,000	11.13	5568.95	363,724.93	5734.63

6010.2



11.13 × 12 × 45 hrs ready hrs / A/C MONTH = 6010.2

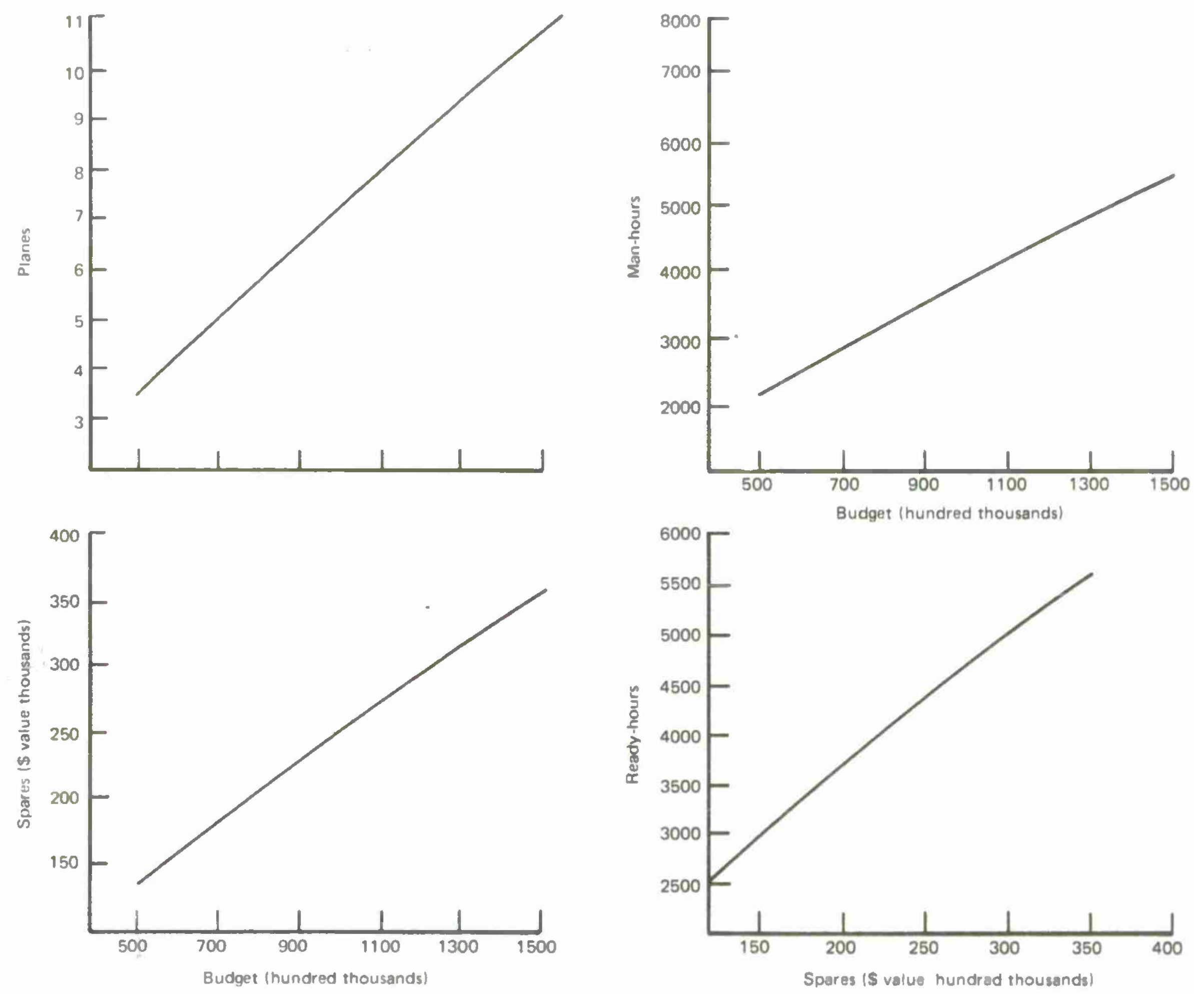


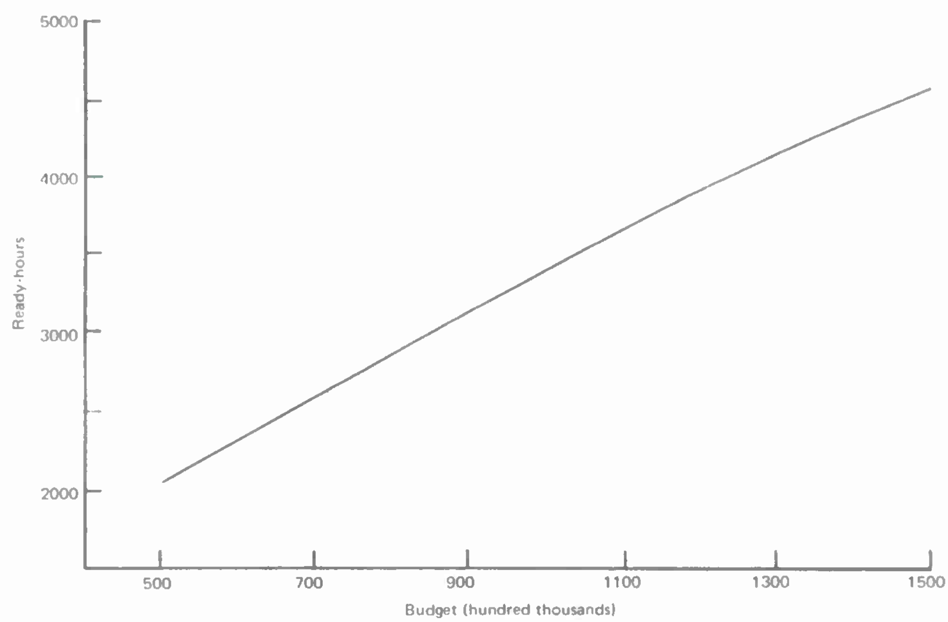
FIG. 6: F-4B RESULTS

TABLE 17

OPTIMAL CH-53 SQUADRONS FOR VARIOUS TOTAL USAGE BUDGET LEVELS

Optimization Results

Cost (dollars)	Planes	Man-hours	Spares (dollars)	Ready hours
500,000	5.47	2834.75	77,898.04	2014.27
550,000	6.02	3134.75	85,738.21	2165.03
600,000	6.56	3436.28	93,582.74	2312.43
650,000	7.11	3739.20	101,431.05	2456.82
700,000	7.65	4043.42	109,283.27	2598 47
750,000	8.20	4348.84	117,138.71	2737.63
800,000	8.74	4655.38	124,997.30	2874 51
850 000	9.29	4963.00	132,858.87	3009.27
900,000	9.83	5271.62	140,723.32	3142.07
950,000	10.38	5581.18	148,590.25	3273.04
1 000 000	10.92	5891.65	156,459.74	3402.31
1,050,000	11.47	6202.98	164,331.60	3529.97
1 100,000	12.01	6515.13	172,205.65	3656.13
1,150,000	12.56	6828.07	180,081.88	3780.85
1,200,000	13.10	7141.76	187,960.14	3904.23
1,250,000	13.64	7456.18	195,840.44	4026.33
1,300,000	14.19	7771.29	203,722.53	4147.21
1 350,000	14.73	8087.07	211,606.42	4266.93
1,400,000	15.28	8403.50	219,492.17	4385.55
1,450,000	15.82	8720.55	227,379.38	4503 10
1,500,000	16.36	9038.21	235,268.40	4619.64



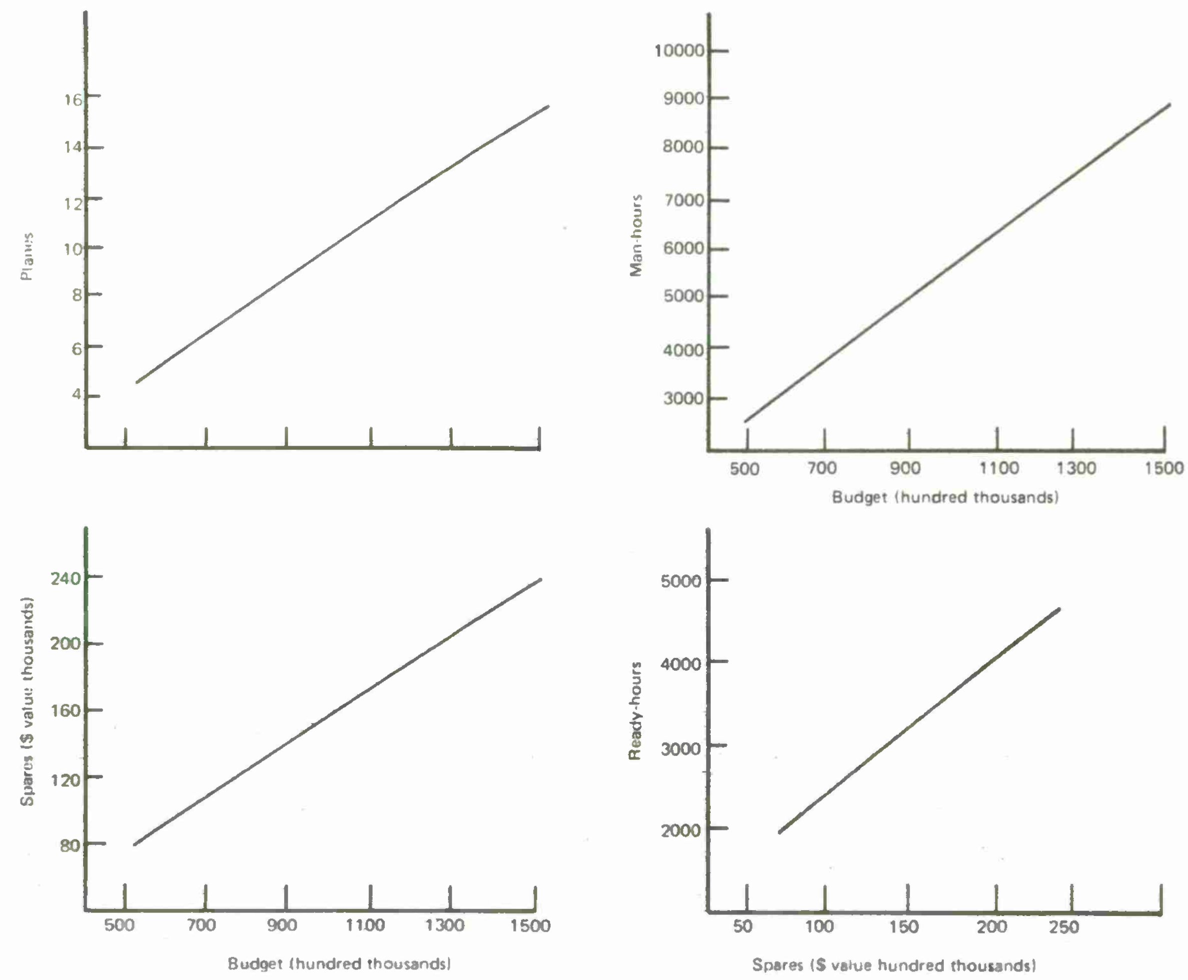


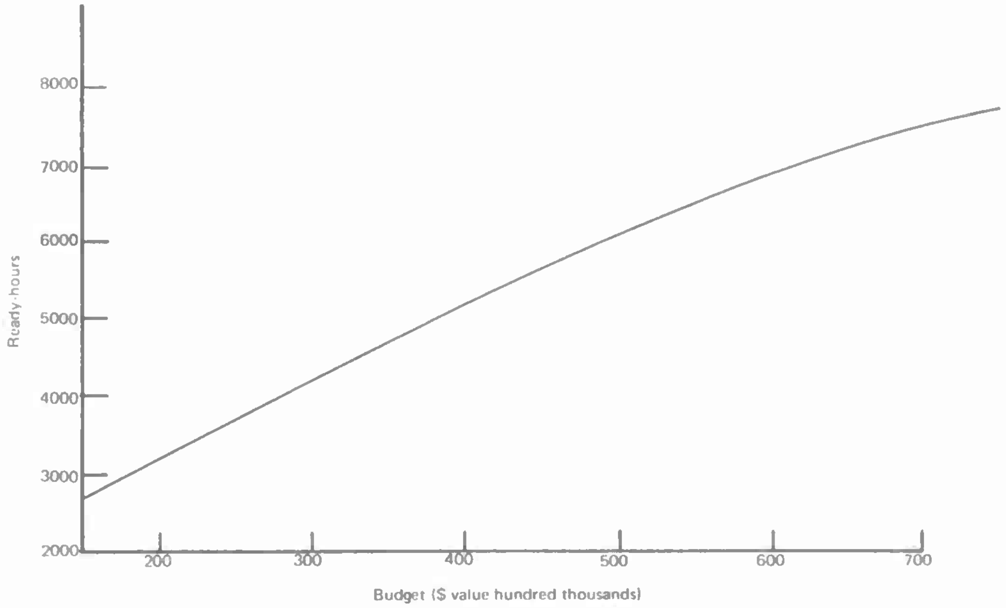
FIG. 7: CH-53 RESULTS

TABLE 18

OPTIMAL TA-4F SQUADRONS FOR VARIOUS TOTAL USAGE BUDGETS

Optimization Results

Cost (dollars)	Planes	Man hours	Spares (dollars)	Ready hours
150,000	6.69	627.21	48,290.75	2650.99
200,000	8.97	849.95	63,688.53	3220.76
250,000	11.25	1075.84	78,936.27	3744.35
300,000	13.54	1304.26	94,064.98	4233.71
350,000	15.84	1534.81	109,095.17	4696.22
400,000	18.14	1767.19	124,041.47	5136.89
450,000	20.45	2001.17	138,914.10	5559.22
500,000	22.76	2236.59	153,721.95	5965.91
550,000	25.07	2473.31	168,471.57	6358.97
600,000	27.38	2711.21	183,168.42	6740.04
650,000	29.70	2950.20	197,816.64	7110.41
700,000	32.03	3190.20	212,420.49	7471.16
750,000	34.35	3431.14	226,983.19	7823.20



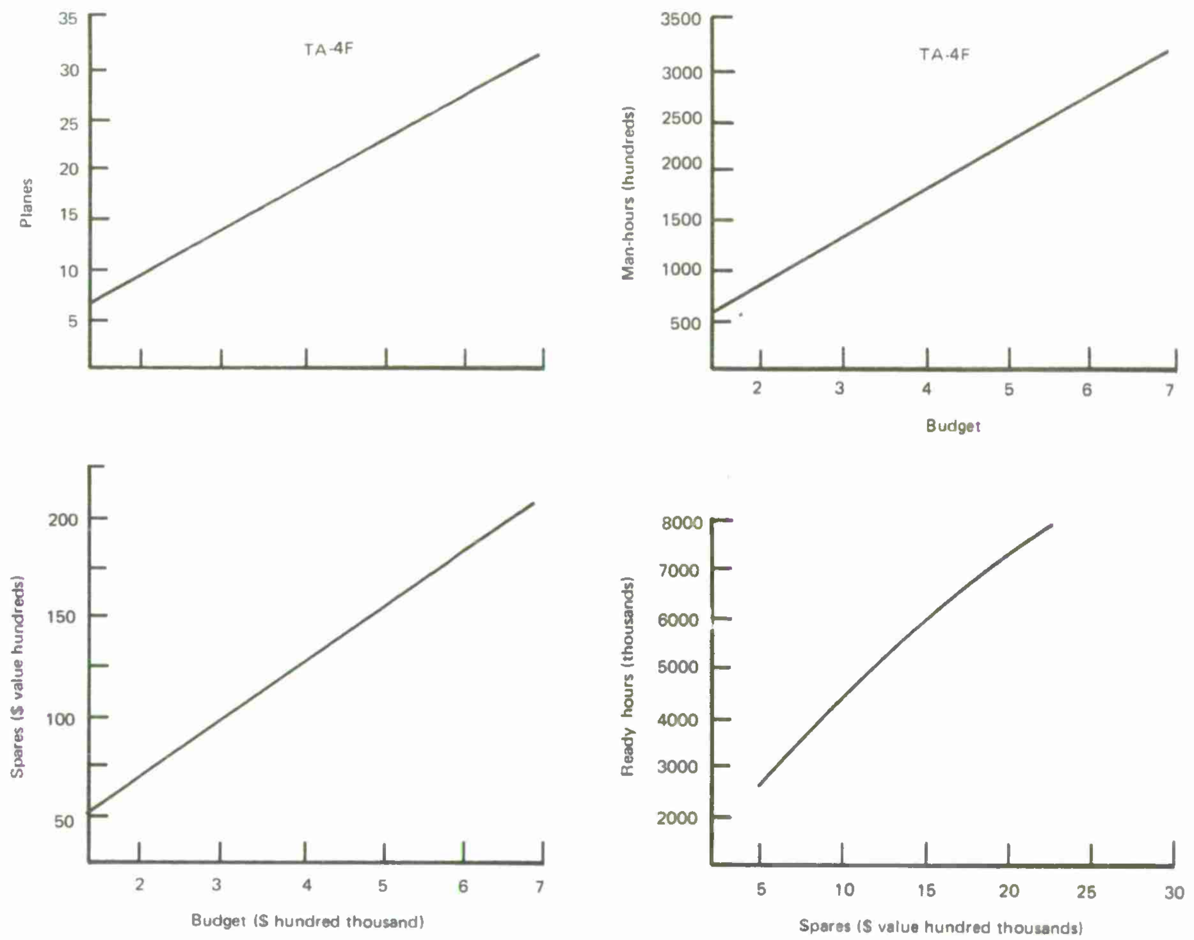


FIG. 8: TA-4F RESULTS

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This study was concerned with the relationship between aircraft readiness and the aircraft, maintenance man-hours, and spare parts available at the squadron level. The objective was to develop and to apply a practical method that could help determine the following things:

1. How aircraft readiness is affected by changes in resource usage, and

2. How to combine these resources to get the highest aircraft readiness for a given budget.

The result desired was a technique that could assist budget planners in estimating the effects on aircraft readiness of changes in the usage for spare parts. The technique was to be illustrated by application to the A-7B, CH-53, S-2E, F-4B, and the TA-4F using data from the aviation 3M data system. This was accomplished by estimating a ready-hour production function and deriving a cost function for the 5 type/model/series of aircraft using the 3M data.

CONCLUSIONS

The important conclusions from this analysis are:

1. The results indicate that the production function methodology can be valuable in examining squadron operations. Much of the observed variance in squadron readiness is explained as a function of resource usage.

2. If current resource budgets are maintained, a larger percentage of these budgets should be allocated to the usage of spare parts at the squadron level. For each type/model/series of aircraft studied, the results showed that higher levels of aircraft ready hours could be obtained.

3. Even if the number of aircraft in a squadron is maintained at current planning levels, the results also indicate a need for more spare parts.

4. For any type/model/series of aircraft the ready-hour production function technique can be used to allocate total resource budgets among the three resources of aircraft, maintenance man-hours, and spare parts for any type/model/series of aircraft in order to achieve maximum ready hour production.

5. Assuming a proportional relationship between spare parts used at the squadron level and dollars budgeted for spares, changes in the spares budget can be related to changes in ready hours and NORS rates by type/model/series of aircraft. The study has not attempted to determine this proportional relationship, however. As noted in Part III of Volume I, to forecast the level of readiness that will be achieved on the average for a given spares support budget requires an estimate of the percentage of the funds allocated that will in fact become available to the user in the form of parts actually required and used. Assuming that management decision processes are unchanged, this percentage may be estimated by determining the dollar value of material actually used in spare part support of the system from the preceding year 3M or ASO demand

data as a percentage of the funds allocated for spare part support of that system in that year. Unfortunately, this estimate was not available for fiscal year 1969, the period for which the production functions were estimated. Fiscal year 1970 is the first year in which funds were allocated to specific weapons systems.

6. There are a large number of possible extensions of the production function technique. Further stratification of the input resource categories could be of even greater use in planning, as trade-offs among various categories of maintenance labor and spare parts could then be considered. Development of an output measure which differentiated among different levels of readiness could be another extension. Finally, extension of the technique to other aircraft types would provide additional input to the budget planning process.

APPENDIX A

APPENDIX A

COSTING METHODOLOGY

This appendix presents the costing methodology and the elements in the costs used in estimating the production function parameters. Specific costs, which are classified, are given in reference (a).

The production function used in this study relates ready hours to the sum of squadron-level and intermediate-level maintenance labor (plus the tools used in maintenance), the number of aircraft in the squadron, and the quantity of spares consumed. The budget equation for the usage of those resources is

$$B = p_1 \cdot M + p_2 \cdot P + p_3 \cdot S ,$$

where M, P, and S are maintenance, planes, and spares, and p_1, p_2, and p_3 are the respective unit costs. These costs would theoretically be the marginal cost (or opportunity cost) of employing one additional unit of that input. The opportunity cost of employing these particular inputs was sufficiently ambiguous to suggest that the costing required additional information. It was decided that the cost estimates would be used to discover whether in future squadrons it would be optimal to have one more (or one less) airplane than there is in existing Navy squadrons. Thus, any conclusions about existing squadrons which are based on these cost estimates should be made cautiously. In particular, the cost of an aircraft in the above equation would probably be a "sunk" cost in existing squadrons.

The variables in the production function are defined only after a period of production is specified. The data on spares consumed had been recorded on a monthly basis. The analysis used spares data as a numeraire, and therefore costs of other inputs were reduced to a monthly number.

The conceptual problems involved with this procedure are revealed by noting that the Navy recently increased the estimates of the operating life of some of the aircraft that were costed. Suppose, for example, that it costs $1000 to purchase an item and after a period of time it is scrapped. If the estimated operating life is 10 months, then the prorated "monthly" cost would be $100; if the operating life is 5 months, the monthly cost would be $200.

Cost estimates were derived using information from standard Service documents. The estimates are average rather than marginal, since the Navy purchases blocs of a particular aircraft rather than just one plane. Neither discount nor inflation "factors" were incorporated, and all costs were in 1970 dollars.

The basis for investment estimates was the unit equivalent (U.E.) aircraft -- that is, on-line aircraft. Costs for the following types were estimated: A-7B, F-4B, CH-53, TA-4F, and S-2E. As typically presented, the investment cost of aircraft contains the following elements:

- Flyaway,
- Investment spares,
- Peculiar ground support equipment.

Estimates of flyaway were obtained from Naval Air Systems Command; investment spares were estimated as a percentage of flyaway cost. Support equipment is included with other maintenance costs. Thus the sum of the first two elements constitutes the cost of one U.E. aircraft. This sum was divided by the aircraft's operating life (which excludes time spent in progressive airframe and other special rework) to obtain a monthly cost. (For 5 of the 6 aircraft, the Navy has recently lengthened the planned operating life: the effect is to reduce the per-month aircraft costs.)

When purchasing U.E. aircraft, we must purchase "support" aircraft, which involve the following investment costs:

- Readiness carrier air wing (RCVW),
- Pipeline,
- Attrition.

If we increase a squadron's U.E. aircraft, we must provide aircraft to train crews for that squadron's mission. For each group of four operating squadrons, there is one readiness (or training) squadron (the RCVW). The ratio of RCVW to U.E. is not always a constant 25 percent, so we used historical averages to obtain our estimates.

Planners in the U.S. Navy appear to assume that the demand for pipeline aircraft is a positive function of the change in the sum of U.E. and RCVW aircraft. This assumption leads to ambiguous implications for this analysis. For example, suppose that in future squadrons the number of aircraft is reduced. Since we are holding output constant, this new policy would imply a higher utilization rate for each remaining aircraft. And as this rate increases, the demand on pipeline facilities (for any given time horizon) should increase. Because of this ambiguity, pipeline aircraft costs (which only include flyaway) are presented separately.

The inclusion of an estimate for attrition aircraft may be the most controversial aspect of this costing exercise. When we examine Navy budget submissions, we can clearly see the allowance for the first two types of support aircraft but no allowance for aircraft that may crash and have to be replaced. However, historical evidence suggests that as the number of operational aircraft (U.E. plus RCVW) increases, the absolute number of aircraft lost will also increase. The Navy publishes estimates of attrition that reflect such factors

as reliability of equipment, type of landing (land or carrier), and type of mission. We used this information to derive the cost of attrition aircraft. (Thus, for this costing exercise, we have assumed that if squadrons reduce their aircraft inventories, the increased utilization rate of the remaining aircraft will not lead to an increase in aircraft fatigue and consequently exhibit a higher attrition rate.)

All the costs discussed thus far were included in the $p_2 \cdot P$ element of the budget equation. A sample illustration may be helpful. The relevant inputs are presented in table A-1.

The first element in the budget equation, $p_1 \cdot P$, is the monthly expenditure on aircraft maintenance and is equal to the sum of the cost of maintenance personnel plus the prorated cost of their equipment (tools, test boxes, etc.). We used data on ground support equipment as a surrogate for data on total maintenance equipment.

Peculiar ground support equipment is probably a function of the number of squadron types and the location of these squadrons. However, support equipment is most often estimated as a percentage of flyaway cost. Several particulars made it difficult to obtain accurate estimates. First, the bulk of support equipment is charged to early "buys." Second, some support equipment may be contained in spare-parts data, since support equipment as well as aircraft may wear out, require repairs, etc.

We used Navy-published data that lists the number of enlisted men by rating required to maintain (at the intermediate and organizational levels) and operate one specific model aircraft for a given time period. From this data we derived the percentage of maintenance time accounted for by men in specific ratings. These percentages were then multiplied by weighted averages that summarize the pay-grade distributions of the ratings.

Our estimates for personnel costs are higher than those contained in the "Navy Program Factors." This is due to the fact that the "Billet Cost Method" that we employed incorporates training costs and rating "survival" rates in order to determine personnel costs.

The cost of maintenance on the CH-53 was estimated in a similar manner. The Marine Corps does not use the Billet Cost Method, so we used standard pay rates published by the Department of Defense. The DoD estimates do not include such elements as training costs and inputted retirement accrual. Even though we attempted to correct these omissions, the final estimates of the costs of Marine Corps personnel are biased downward relative to those of the Navy.

TABLE A-1

INPUTS FOR ESTIMATING AIRCRAFT INVESTMENT COSTS
(Hypothetical)

Row	Element	Input
1	Type aircraft	F-1A
2	Flyaway cost	\$2.2 M
3	Investment spares (5% of row 2)	\$110,000
4	Operating life	100 months
5	Prorated flyaway cost (row 2 divided by row 4)	\$22,000
6	U.E. monthly cost (sum of rows 2 + 3 divided by row 4)	\$23,100
7	RCVW factor	25 percent
8	Pipeline factor	20 percent
9	Attrition factor	0.3 percent/month

Step 1

The monthly cost of RCVW aircraft is (0.25) x (\$23,100) = \$5,775 .

Step 2

The monthly cost of operating aircraft (U.E. + RCVW) is (\$23,000) + (\$5,775) = \$28,775 .

Step 3

The cost of pipeline aircraft is obtained by first multiplying the pipeline factor by the operating aircraft, then multiplying this result by flyaway cost. The monthly cost is (0.20) x (1.25) = 0.25 ,

(0.25) x (\$22,000) = \$5,500 .

Step 4

The cost of attrition aircraft is obtained by first multiplying the attrition factor by the operating aircraft, then multiplying this result by flyaway cost. The monthly cost is (0.003) x (1.25) = 0.004 ,

(0.004) x (\$22,000) = \$88 .

Step 5

The total monthly cost of $p_2 \cdot P$ in the budget equation is thus

U.E. + RCVW	\$ 28,775
Pipeline	5,500
Attrition	88
Total	\$ 34,363/month

Note that this last total consists of prorated investment costs and excludes operating costs.

Maintenance costs are a subcategory of general operating costs. The approach of the study group was to hold output constant while varying the input mix. This procedure implies that some inputs, e.g., petroleum and number of pilots in a squadron, could also be considered constants. Hence, costs of these latter resources were not estimated.

Finally, it is noted that various specifications of these costs can be considered reasonable, and thus, in future use of the methodology, more attention should be directed to the costing problem. Empirical results of the type presented in this volume can easily be obtained for any set of cost data.

REFERENCE

(a) "Costing in Support of INS Study 32: A Study of Aviation Spares and Readiness Relationships," Unpublished manuscript.

APPENDIX B

APPENDIX B

SUMMARY OF COMPUTER OPERATIONS

INTRODUCTION

This appendix describes the data processing used in this study. A general description of the computer techniques used to evaluate the production and cost functions is also presented. A macro-flow chart of the data processing system developed is included in section III.

SECTION I. DATA PROCESSING

A data bank of aviation, maintenance, and material information on the examined aircraft in this study was procured from the Navy Maintenance Material Management System (3-M files). This system records man-hour, maintenance, and aircraft statistical data related to an operating unit on a day-to-day basis. Three files from this system were utilized: (1) 3-M Maintenance Action - TID No. 8, PAR 12/581; (2) 3-M Material - 60 series - TID No. 8, PAR 12/634; and (3) ASD data - 3-M Readiness, Flight NOR - TID No. 8, PAR 12/655.

Data Extraction

It was necessary to extract the data for each aircraft onto a separate file, since the 5 aircraft types were aggregated on the 3-M files. The aircraft type was identified by its unique "type equipment codes," shown in the following table:

Aircraft	Type equipment codes
A-7B	AAFB
F-4B	AFPB, AFPC
S-2E	ASAJ
CH-53	AHXA
TA-4F	AACF

Programs EXTMAF, EXTMAT, and EXTNOR were written to separate aircraft types from the maintenance, material, and ASD files, respectively. These extraction programs examined the type equipment code, the organizational code (squadron), and action date. Any record where the organization code was missing or invalid or where the action date was not in the 18-month time period studied was deleted. Type equipment codes for the aircraft to be extracted were input. The formats of the resultant MAF, MAT and NOR files were identical to their corresponding input files.

Data Summarization

Data on the individual aircraft were summarized by squadron, employed within each of the 18 months studied. The squadron was identified by the 3-digit alpha-numeric organization code found on each file. Since the data was reported on a day-to-day basis, all summary points were aggregated by month. The following data was summarized for each separate squadron appearing in a month:

A. Maintenance Action File (MAF)

1. Date - Julian action date
2. Squadron – organizational code
3. Number of items and maintenance man-hours required by the squadron. Each entry on the MAF file represented one maintenance action. The level of the action could be squadron (organization) or intermediate. Organizational maintenance refers to those maintenance functions normally performed by an operating unit in support of its own operations. Intermediate maintenance refers to those maintenance functions normally performed in centrally located facilities for the support of operating units either aboard ship, or at a particular station, or within a designated area. Record code LVL identified this level, where a "1" indicated organization and a "2" intermediate. The record represented a cannibalization activity when record code AT was a "T" or "U". The man-hours expended and number of items required for maintenance were summarized:

   a. At the organization level
   b. At the intermediate maintenance level
   c. On cannibalization activities

B. Material File (MAT)

1. Date – Action date, month, and year
2. Squadron – organization code
3. Parts usage by the squadron. A record on the MAT file could be distinguished as a repairable parts entry when record code MAT CTL contained the alpha character "G," "H," or "Q". Otherwise the record was a consumable parts entry. This quantity and price of parts was summarized by:

   a. Repairable parts
   b. Consumable parts

4. Total price (spares). This cost was calculated as 20 percent of the price of repairable parts plus the price of consumables.

C. ASD Data (NORS)

1. Date - month and year
2. Squadron - action organization
3. Number of aircraft available to the squadron. This was defined as the number of Bureau Numbers on which actions were performed by the appropriate squadron.
4. Hours in the NORM (maintenance) condition
   a. Scheduled maintenance
   b. Unscheduled maintenance
5. NORS hours. This is the number of hours during which the aircraft was not operationally ready.
6. Flights
   a. Number
   b. Hours
7. Custodial hours. This is the number of hours the aircraft was in ready reporting status.
8. Ready hours. This was computed as Custodial Hours minus NORS, scheduled, and unscheduled NORM hours.

In addition to summary reports, the programs SUMMAF, SUMMAT, and SUMNOR produced a new aircraft data tape for each of the 3 files examined. Each individual squadron within a month and its corresponding summary data points constituted a record. Since the analysis proceeded on the aviation data in these summary tapes, the record formats are presented in section III.

Data Stratification. The three summary tapes MAT, MAF and NOR were merged by matching records on squadron within month. When a particular squadron-month entry was not found on all 3 files, the appropriate records were deleted. The program produced a listing of all matched records and an aircraft summary tape. The record format of this tape is found in Section III.

At this point it was desired to stratify the aircraft summary date base by location and deployment. Atlantic and Pacific squadrons were separated. For the A-7B, F-4B, S-2E, and TA-4F aircraft, an "A" as the first character of the squadron code distinguished an Atlantic squadron and a "P" a Pacific squadron. For the CH-53 aircraft, an "F" distinguished an Atlantic Squadron and a "G" a Pacific squadron. Furthermore, training squadrons were separated when appropriate for the aircraft type. It was also necessary in some cases to delete a squadron

whose associated data points were obviously erroneous. Various short programs were written to accomplish these tasks. In all cases a binary output tape of 22 words per record was produced. These 22 variables corresponded to the aircraft summary data points found in Section III with the month and squadron identification deleted. Stratifications of each aircraft data base were stacked as separate files on one tape.

SECTION II. PRODUCTION AND COST FUNCTION ANALYSES

This section presents a brief description of the computer techniques employed to evaluate the 2-input production functions and the cost function discussed in this volume.

Cobb-Douglas Production Function

The Cobb-Douglas form of the production function could be treated as a single equation that is linear in the unknown parameters. Thus the parameters could be estimated by the method of least squares. A multiple-regression program developed at the University of Chicago was employed to evaluate this function. This program, BIMED34, calculates multiple linear regression in a step-wise manner. That is, the program re-examines at every stage of the regression the variables incorporated into the model in previous stages. Because the program performs transformations (e.g., LOGF(X_1)) and selection of variables, the existing binary data bases for each aircraft were input to this program.

Constant Elasticity of Substitution (C. E. S.) Production Function

A program to estimate the parameters of the non-linear C. E. S. production function was developed. A system of 8 simultaneous non-linear equations in the 8 unknown parameters was formed by setting the partial derivatives of the C. E. S. function to zero. A system of linear equations was provided by expanding each of the non-linear equations about assumed initial values of the parameters. The program iteratively solved this set of equations for a new set of parameter estimates. Observed ready hours, maintenance man-hours, number of planes and spares from the aircraft summary data base, and the initial estimates of the unknown parameters were input.

The solution technique used the Newton-Raphson iteration procedure, which computes successive approximations to the desired roots of the equations until the system converges.

The general solution form of the Newton-Raphson method is

$$\Delta B = (\Delta R)^{-1} R ,$$

where r_{ij}, an element of the matrix ΔR, is the second partial derivative of the function with respect to B_i and B_j. The components of the vector R are the first derivatives of the function with respect to each parameter.

Each component of ΔR and R is evaluated at the point B_0, a set of initial parameter estimates. ΔB, the parameter-change vector, is then calculated, and a new set of parameter estimates is obtained by adding ΔB to B_0. Each iteration reevaluates the partial derivatives at the new point, and a new parameter-change vector is calculated. This process continues until the parameter changes are sufficiently small.

The complexity of this particular set of equations generated problems in the numerical stability of the solution. Round-off errors occurred in the inversion of R and provoked oscillation in ΔB in successive iterations. Thus it was necessary to incorporate an alternate convergence criteria. Ready hours were predicted by a direct computation of the C. E. S. function by substituting the new parameter estimates into the formula. A multiple correlation coefficient was computed between the predicted and observed ready hours. When the multiple correlation coefficient failed to increase appreciably between successive iterations, convergence was assumed.

Cost Function

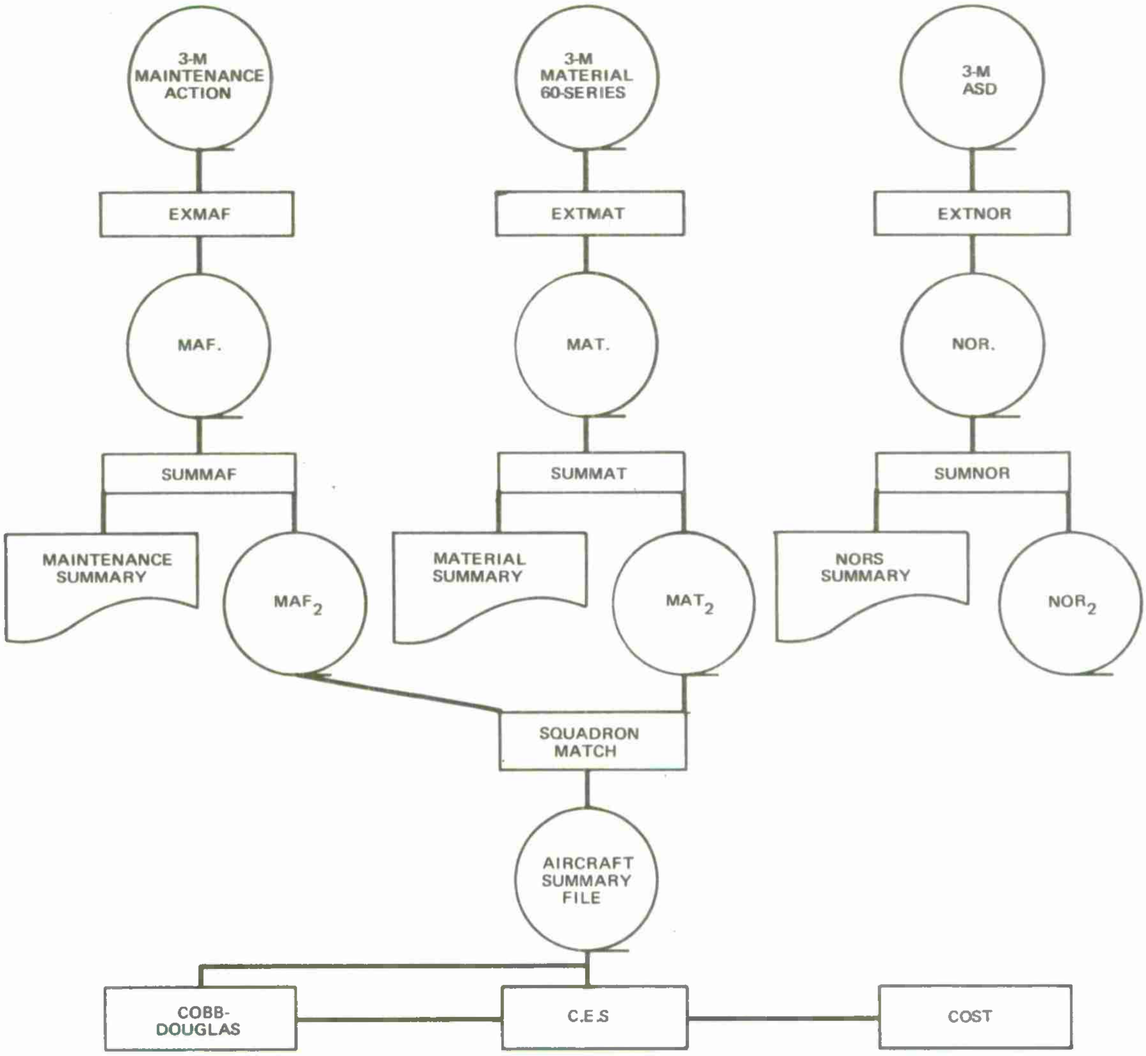
A program was written to determine the optimal mixture of the resources to be supplied to a squadron given various funding levels for resource usage. The non-linear optimization model used in this analysis is presented in the results of the cost function analysis section of this volume.

The parameter estimates derived from the production function, the per-unit prices of the resources - man-hours, spares and planes - and a range of funding levels for resource usage were input. The program solved production function equation for the optimal level of planes (P*) through an iterative procedure initial estimate of the optimal number of planes (P*) was input, and the equation was directly solved for C (funding level). The estimate of P* was incrementally adjusted, and the equation reevaluated until the value of C was equal to the funding level desired. Given this optimal value of planes, the optimal values of man-hours (M*) and spare parts usage (S*) were determined through the direct solution of equations (2) and (3) on page 20. This procedure was continued for each of the funding levels desired.

SECTION III. SYSTEM ILLUSTRATIONS AND AIRCRAFT SUMMARY FILE FORMATS

This section includes on the following pages a Macro-flowchart of the data processing system and formats of the aircraft summary files for illustrative purposes.

MACRO–FLOWCHART FOR PROCESSING OF EACH AIRCRAFT TYPE



MAF DATA SUMMARY RECORD FORMAT

Columns	Format	Description
1-2	I2	Month (1-18)
3-5	A3	Squadron
6-13	I8	No. jobs at squadron level
14-21	I8	No. jobs at intermediate level
22-29	I8	No. jobs activity cannibalization
30-37	I8	Total no. jobs
38-47	F10.0	Maintenance man-hours at squadron level
48-57	F10.0	Maintenance man-hours at intermediate level
58-67	F10.0	Cannibalization hours
68-77	F10.0	Total hours

MAT DATA SUMMARY RECORD FORMAT

Columns	Format	Description
1-2	I2	Month (1-18)
3-5	A3	Squadron
6-17	F12.2	No. repairable parts
18-29	F12.2	Price repairable parts
30-41	F12.2	No. cannibalization parts
42-53	F12.2	Price cannibalization parts
54-65	F12.2	Total no. parts
66-77	F12.2	Total price of parts

NOR DATA SUMMARY RECORD FORMAT

Columns	Format	Description
1-2	I2	Month (1-18)
3-5	A3	Squadron
6-15	I10	Scheduled NORM hours
16-25	I10	Unscheduled NORM hours
26-35	I10	NORS hours
36-45	I10	Number of flights
46-55	I10	Flight hours
56-65	I10	Custodial hours
66-75	I10	Ready hours
76-85	I10	Number of planes

AIRCRAFT SUMMARY FILE FORMAT

Columns	Format	Description
	Record 1	
1-2	I2	Month
3-5	A3	Squadron
6-19	F14.2	Scheduled NORM hours
20-33	F14.2	Unscheduled NORM hours
34-47	F14.2	NORS hours
48-61	F14.2	Number of flights
62-75	F14.2	Flight hours
76-89	F14.2	Custodial hours
90-103	F14.2	Ready hours
104-117	F14.2	Number of planes
	Record 2	
1-14	F14.2	No. repairable parts
15-28	F14.2	Price repairable parts
29-42	F14.2	No. cannibalization parts
43-66	F14.2	Price cannibalization parts
67-80	F14.2	Total no. parts
81-94	F14.2	Total price of parts
	Record 3	
1-14	F14.2	No. jobs at squadron level
15-28	F14.2	No. jobs at intermediate level
29-42	F14.2	No. jobs-cannibalization activity
43-66	F14.2	Total no. jobs
67-80	F14.2	Hours at squadron level
81-94	F14.2	Hours at intermediate level
95-108	F14.2	Cannibalization hours
109-122	F14.2	Total hours